FACTOR AUGMENTED AUTOREGRESSIVE DISTRIBUTED LAG MODELS WITH MACROECONOMIC APPLICATIONS

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Abstract

This paper proposes a factor augmented autoregressive distributed lag (FADL) framework for analyzing the dynamic effects of common and idiosyncratic shocks. We first estimate the common shocks from a large panel of data with a strong factor structure. Impulse responses are then obtained from an autoregression, augmented with a distributed lag of the estimated common shocks. The approach has three distinctive features. First, identification restrictions, especially those based on recursive or block recursive ordering, are very easy to impose. Second, the dynamic response to the common shocks can be constructed for variables not necessarily in the panel. Third, the restrictions imposed by the factor model can be tested. The relation to other identification schemes used in the FAVAR literature is discussed. The methodology is used to study the effects of monetary policy and news shocks.

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1 Introduction

This paper proposes a new approach for analyzing the dynamic effects of q common shocks such as due to monetary policy and technology on q or more observables. We assume that a large panel of data $X^{ALL} = (X, X^{OTH})$ is available and use the sub-panel X that is likely to have a strong factor structure to estimate the common shocks. Identification is based on restrictions on a q dimensional subset of X or on q block specific dynamic shocks. The impulse response coefficients are obtained from an autoregression in each variable of interest augmented with current and lagged values of the identified common shocks. Observed factors can coexist with latent factors. We refer to this approach as Factor Augmented Autoregressive Distributed Lag (FADL).

An important feature of the FADL is that it estimates the impulse responses using minimal restrictions from the factor model. The approach has several advantages. First, while X is large in dimension, identification is based on a subset of variables whose dimension is the number of common shocks. This reduces the impact of invalid restrictions on variables that are not of direct interest. Second, the impulse responses are the coefficients estimated from a regression with common shocks as predictors. Additional restrictions are easy to impose, and for many problems the impulse responses can be estimated on an equation by equation basis. Third, the analysis only requires a strong factor structure to hold in X and is less likely to be affected by the possibility of weak factors in X^{OTH}.

The proposed FADL methodology lets the data speak whenever possible and is in the spirit of vector-autoregressions (VAR) proposed by Sims (1980). The FADL also shares some similarities with the Factor Augmented Vector Autoregressions (FAVAR) considered in Bernanke and Boivin (2003). Their FAVAR expands the econometrician's information set without significantly increasing the dimension of the system. Our FADL further simplifies the analysis by imposing restrictions only on the variables of interest. Recursive and non-recursive restrictions can be easily implemented.

The FADL is derived from a structural dynamic factor model which has a restricted FAVAR as its reduced form. A factor model imposes specific assumptions on the covariance structure of the data. Even though many variables are available for analysis, a factor structure may not be appropriate for every series. As noted in Boivin and Ng (2006), more data may not be beneficial for factor analysis if the additional data are noisy and/or do not satisfy the restrictions of the factor model. We treat X like a training sample. Using it to estimate the common shocks enables us to validate the factor structure in X^{OTH}, the series not in X.

The FADL approach stands in contrast to structural FAVARs that impose all restrictions of a dynamic factor model in estimation, as Forni, Giannone, Lippi, and Reichlin (2009). The FADL estimates will necessarily be less efficient if the restrictions are correct, but are more robust when the restrictions do not hold universally. As in Stock and Watson (2005), our FADL also permits implications of the factor model to be tested. However, we go one-step further by letting the data determine the Wold representation instead of inverting a large FAVAR.

The paper proceeds as follows. Section 2 first sets up the problem of identifying the effects of common shocks from the perspective of a dynamic factor model. It then presents the FADL framework without observed factors. Estimation and identification of a FADL is discussed in Section

3. Relation of FADL to alternative structural dynamic factor analysis is discussed in Section 4, and FADL is extended to allow for observed factors. Simulations are presented in Section 5. Section 6 considers the identification of monetary and news shocks and the estimation of their effects on economic activity. Both examples highlight the main features of FADL: the ability to perform impulse responses analysis and to test the validity of the factor structure of variables not used in estimation or identification of the common shocks.

2 Dynamic Factor Models and the FADL Framework

Let N be the number of cross-section units and T be the number of time series observations where N and T are both large. We observe data $X^{ALL} = (X,\ X^{OTH})$ which are stationary or have been transformed to be covariance stationary. It is assumed that $X_t = (X_{1t}, \ldots, X_{Nt})'$ has a (strong) factor representation and can be decomposed into a common and an idiosyncratic component:

$$X_t = \widetilde{\lambda}(L) f_t + u_{Xt} \tag{1}$$

where $f_t = (f_{1t}, \ldots, f_{qt})'$ is a vector of q common factors and $\widetilde{\lambda}(L) = \widetilde{\lambda}_0 + \widetilde{\lambda}_1 L + \ldots \widetilde{\lambda}_s L^s$ is a polynomial matrix of factor loadings in which the $N \times q$ matrix $\widetilde{\lambda}_j = (\widetilde{\lambda}_{j1} \ldots, \widetilde{\lambda}_{jN})'$ quantifies the effect of the common factors at lag j on X_t. In particular, the i-th element of $\widetilde{\lambda}(L)$ consists of $\widetilde{\lambda}_i(L) = (\widetilde{\lambda}_{i1}(L), \ldots, \widetilde{\lambda}_{iq}(L))$, $\widetilde{\lambda}_{ik}(L) = \sum_{j=0}^{p_{i,k}} \widetilde{\lambda}_{i,k,j} L^j$, where $p_{i,k}$ is the finite lag order of the $k-$th factor loadings polynomial of variable i. The series-specific errors $u_{Xt} = (u_{X1t}, \ldots, u_{XNt})'$ are mutually uncorrelated but can be serially correlated. We assume

$$u_{Xt} = D(L) u_{X,t-1} + v_{Xt} \tag{2}$$

where $D(L)$ is a finite-order N-dimensional matrix poynomial and v_{Xt} is a vector white noise process. The q latent dynamic factors are assumed to be a vector autoregressive process of order h. Without loss of generality, we assume $h = 1$ and thus

$$f_t = \Gamma_1 f_{t-1} + \Gamma_0 v_{ft} \tag{3}$$

where the characteristic roots of $I_q - \Gamma_1 L$ are strictly less than one. The $q \times 1$ vector v_{ft} consists of structural common shocks (such as monetary policy or technology). These structural shocks can affect several dynamic factors simultaneously. Hence, the $q \times q$ matrix Γ_0 need not be an identity. By assumption, $E(v_{Xit} v_{Xjt}) = 0$ and $E(v_{Xit} v_{fkt}) = 0$ for all $i \neq j$ and for all $i = 1, \ldots N$ and $k = 1, \ldots q$.

Assuming that $I - D(L)L$ is invertible, the vector-moving average representation of X_t in terms of the structural common and idiosyncratic shocks is

$$X_t = \Psi^f(L) v_{ft} + \Psi^X(L) v_{Xt}.$$

The structural impulse response coefficients Ψ_j^X and Ψ_j^f are defined from

$$\begin{aligned}
\Psi^X(L) &= \sum_{j=0}^{\infty} \Psi_j^X L^j = (I - D(L)L)^{-1} \\
\Psi^f(L) &= \sum_{j=0}^{\infty} \Psi_j^f L^j = \widetilde{\lambda}(L)(I - \Gamma_1 L)^{-1} \Gamma_0.
\end{aligned}$$

For each $j \geq 0$, Ψ_j^f is a $N \times q$ matrix summarizing the effect of a unit increase in v_{ft} after j periods. We use $\Psi^f_{j,i_1:i_2,k_1:k2}$ to denote the submatrix in the i_1 to i_2 rows and k_1 to k_2 columns of Ψ_j^f. When $i_1 = i_2 = i$ and $k_1 = k_2 = k$, we use $\psi^f_{j,i,k}$ to denote the effect of shock k in period t on series i in period $t + j$.

The objective of the exercise is to uncover the dynamic effects (or the impulse response) of the structural common shocks v_{ft} on variables of interest. By using $X_{1t}, \dots, X_{Nt}$ for factor analysis, the econometrician's information set is of dimension N. Forni, Giannone, Lippi, and Reichlin (2009) argue that non-fundamentalness is generic of small scale models but cannot arise in a large dimensional dynamic factor model. The reason is that $\Psi^f(z)$ is a rectangular rather than a square matrix and its rank is less than q for some z only if all $q \times q$ sub-matrices of $\Psi^f(z)$ are singular, which is highly unlikely. Assuming that N is large ensures that the common shocks are fundamental for X.

However, even if N is large, nothing distinguishes one common shock from another. In a VAR analysis with q endogenous variables and q shocks, $q(q-1)/2$ restrictions will be necessary. A popular approach is to impose contemporaneous exclusion restrictions such that a rank condition is satisfied, see, eg. Deistler (1976), Rubio-Ramírez, Waggoner, and Zha (2010). If the identification restrictions imply a recursive ordering, then the parameters can be identified sequentially and estimation can proceed on an equation by equation basis.

While $\Psi_0^X = I_N$ in a dynamic factor model, the contemporaneous response of X_t to common shocks v_{ft} is given by

$$\Psi_0^f = \widetilde{\lambda}_0 \Gamma_0 = \begin{pmatrix} \widetilde{\lambda}_{0,1,1} & \widetilde{\lambda}_{0,1,2} & \dots & \widetilde{\lambda}_{0,1,q} \\ \vdots & & & \\ & \vdots & & \\ \widetilde{\lambda}_{0,q,1} & \widetilde{\lambda}_{0,q,2} & \dots & \widetilde{\lambda}_{0,q,q} \\ \vdots & & \vdots & \\ \widetilde{\lambda}_{0,N,1} & \widetilde{\lambda}_{0,N,2} & \dots & \widetilde{\lambda}_{0,N,q} \end{pmatrix} \begin{pmatrix} \Gamma_{0,1,1} & \dots & \Gamma_{0,1,q} \\ \vdots & & \\ \Gamma_{0,q,1} & \dots & \Gamma_{0,q,q} \end{pmatrix}.$$

The (i,k) entry of $\widetilde{\lambda}_0$ is the contemporaneous effect of factor k on series i, and the (k,j) entry of Γ_0 is the effect of the j-th common shock on factor k. In general, Ψ_0^f will not be an identity matrix.

Two additional problems make the identification problem non-standard. First, while having more total shocks than endogenous variables should facilitate identification, the common shocks also restrict the co-movements across series. Imposing constraints on an isolated number of series is actually quite difficult within the factor framework. Zero restrictions on the entries of λ_0 or Γ_0 alone are not usually enough to ensure that a particular entry of Ψ_0^f takes on the desired value (often zero). Second, the dynamic factors are themselves latent. Thus, not only do we need to identify the effects of v_f, we also need to identify v_f.

Our analysis is based on the following assumptions.

Assumption 1: $E(v_{ft}) = 0, \quad E(v_{ft} v_{ft}') = I_q$.

Assumption 2: $D(L)$ is a diagonal matrix with $\delta_i(L)$, a finite $p_{x,i}$-degree lag polynomial, in the i-th diagonal, i.e.

$$D(L) = \begin{pmatrix} \delta_1(L) & 0 & \dots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \dots & \delta_N(L) \end{pmatrix},$$

Assumption 3: For some j, a $q \times q$ matrix of Ψ_j^f is full rank.

Assumption 1 is a normalization restriction as we cannot separate the size of the common shocks from their impact effects. Assumption 2 is a form of exclusion restriction. We assume univariate autoregressive dynamics for idiosyncratic errors:

$$u_{Xit} = \delta_i(L)u_{Xit-1} + v_{Xit}.$$

This implies that dynamic correlations between any two series are due entirely to the common factors, which is the defining feature of a dynamic factor model. Diagonality of $D(L)$ in turn allows X_{it} to be characterized by an autoregressive distributed lag model with serially uncorrelated idiosyncratic errors:

$$X_{it} = \delta_i(L)X_{it-1} + \lambda_i(L)f_t + v_{Xit}, \tag{4}$$

where $\lambda_i(L) = (1 - \delta_i(L))\widetilde{\lambda}_i(L)$. For simplicity, we assume $\lambda_i(L)$ is of order s for all i.[1] A representation that is more useful for impulse response analysis is an autoregressive distributed lag in the primitive shocks v_{ft}:

$$X_{it} = \delta_i(L)X_{it-1} + \psi_i^f(L)v_{ft} + v_{Xit} \tag{5}$$

where

$$\psi_i^f(L) = (1 - \delta_i(L))\widetilde{\lambda}_i(L)(I - \Gamma_1 L)^{-1}\Gamma_0.$$

We will henceforth refer to (5) as the FADL representation of X_{it}. Note that $(1 - \delta_i(L))^{-1}\psi_i^f(L)$ is precisely the i-th row of $\Psi^f(L)$.[2]

The dynamic effects of the common shocks v_{ft} on X_{it} are defined by the coefficients $\psi_i^f(L)$. If v_f were observed and $N = q$, equation (5) defines a dynamic simultaneous equations system in which identification can be achieved by excluding some v_f or its lags from certain equations. For example, contemporaneous restrictions can be imposed so that the $q \times q$ matrix Ψ_0^f has rank q. As

[1] In general, $\lambda_i(L)$ is a lag polynomial of the maximum order $\max_{k \in 1,\dots,q}(p_{i,k})$ times $p_{i,x}$.

[2] In Dufour and Stevanovic (2013) the authors argue that factors' dynamics obey, in general, a VARMA process. The FADL representation still holds. Assume an identified VARMA(1,1) process for factors:

$$f_t = \Gamma_1 f_{t-1} + \Theta_1\Gamma_0 v_{ft-1} + \Gamma_0 v_{ft}.$$

Then, the FADL representation of X_{it} is the same as in (5) except that

$$\psi_i^f(L) = (1 - \delta_i(L))\widetilde{\lambda}_i(L)(I - \Gamma_1 L)^{-1}(I + \Theta_1)\Gamma_0.$$

our system is tall with $N \geq q$, Assumption 3 requires that a $q \times q$ submatrix of Ψ_j^f is full rank. If all restrictions are imposed on Ψ_0^f, Assumption 3 will hold if the top $q \times q$ submatrix of Ψ_0^f has rank q. However, long run and sign restrictions are also permitted.

Assumptions 1 to 3 are fairly standard. But our factors are also latent and we can only identify the space spanned by the factors and not the factors themselves. To make the procedure operational, we need to replace v_{ft} by estimates $\widehat{v}_{ft}$ which have the same properties as Assumption 1. These identification conditions will be further developed below.

3 Estimation and Identification

If there are q common shocks, we will need at least q series for identification. Without loss of generality, let Y_t be the first q series in X_t. Since each $y_t \subset Y_t$ admits a dynamic factor structure, it holds that

$$y_t = \alpha_{yy}^*(L)y_{t-1} + \alpha_{yf}^*(L)v_{ft} + v_{yt}^*, \tag{6}$$

where $\alpha_{yy}^*(L)$ and $\alpha_{yf}^*(L)$ are the autoregressive and distributed lag coefficients of the non feasible regression since the common shocks v_{ft} are, in general, not observable. Our impulse response analysis is rather based on least squares estimation of the FADL

$$y_t = \alpha_{yy}(L)y_{t-1} + \alpha_{yf}(L)\widehat{v}_{ft} + v_{yt} \tag{7}$$

where a prior restrictions are be imposed on $\alpha_{yf}(L)$ for identification. We now explain how v_{ft} is estimated and how restrictions are imposed on the FADL.

Let Λ be the $N \times r$ matrix of loadings, F_t be a $r = q(s+1) \times 1$ vector of static factors, where

$$\Lambda = \begin{pmatrix} \Lambda_1 \\ \Lambda_2 \\ \vdots \\ \Lambda_N \end{pmatrix}, \qquad F_t = \begin{pmatrix} f_t \\ f_{t-1} \\ \vdots \\ f_{t-s} \end{pmatrix}, \qquad \Phi_F = \begin{pmatrix} \Gamma_1 & \Gamma_2 & . & . & . & \Gamma_s \\ I_q & 0 & . & 0 & . & 0 \\ 0 & I_q & & 0 & . & 0 \\ . & 0 & & . & . & 0 \\ 0 & 0 & . & I_q & . & 0 \end{pmatrix} \qquad \Lambda_i = \begin{pmatrix} \lambda_{i0} & \lambda_{i1} & \ldots \lambda_{is} \end{pmatrix}.$$

Note that we have assumed a VAR(1) process for dynamic factors. Despite it implies a particular structure for F_t and Ψ_F, there is no loss of generality.[3] The starting point is the static factor representation of the pre-whitened data, $x_{it} = (1 - \delta_i(L)L)X_{it}$:

$$x_{it} = \Lambda_i F_t + v_{Xit} \tag{8}$$

$$F_t = \Phi_F F_{t-1} + \varepsilon_{Ft} \tag{9}$$

$$\varepsilon_{Ft} = G\varepsilon_{ft}. \tag{10}$$

[3] Bai and Ng (2007) show that the vector of static factors, F_t, is of dimension $r = q(s+1) \times 1$ and follows a VAR of order depending on h and s. For instance, if f_t follows a finite-order VAR(h), then $F_t = (f_t', \ f_{t-1}', \ldots, f_{t-\max(h,s)}')'$ follows a VAR with

$$\Phi_F = \begin{pmatrix} \Gamma_1 & \Gamma_2 & . & . & . & \Gamma_{\max(h,s)+1} \\ I_q & 0 & . & 0 & . & 0 \\ 0 & I_q & & 0 & . & 0 \\ . & 0 & & . & . & 0 \\ 0 & 0 & . & I_q & . & 0 \end{pmatrix}$$

where $\Gamma_{h+1} = \ldots = \Gamma_{\max(h,s)+1} = 0$.

The ε_{Ft} are the reduced form errors of F_t and are themselves linear combinations of the structural shocks v_{ft} and $\varepsilon_{ft} = \Gamma_0 v_{ft}$ is the vector of reduced form common shocks, see (3). The $r \times q$ matrix G maps the structural dynamic shocks to the reduced form static shocks. Since X_t is assumed to have a strong factor structure, $\Lambda'\Lambda/N \to \Lambda > 0$ as $N \to \infty$, and the $N \times N$ matrix $\frac{1}{T}\sum_{t=1}^{T} x_t x_t'$ has r eigenvalues that diverge as $N, T \to \infty$ while the largest eigenvalue of the $N \times N$ covariance matrix of v_{Xt} is bounded.

From $v_{Xit} = x_{it} - \Lambda_i F_t = x_{it} - \Lambda_i(\Phi_F F_{t-1} + \varepsilon_{Ft})$, define

$$\begin{aligned} \varepsilon_{Xit} &= x_{it} - \Lambda_i \Phi_F F_{t-1} \\ &= \Lambda_i \varepsilon_{Ft} + v_{Xit}. \end{aligned} \tag{11}$$

As noted in Stock and Watson (2005), the rank of the $r \times 1$ vector ε_{Ft} is only q, since F_t is generated by q common shocks. In other words, ε_{Xit} itself has a factor structure with common factors ε_{ft}. But ε_{ft} are themselves linear combinations of v_{ft}. Let

$$v_{ft} = \widetilde{B}\varepsilon_{ft},$$

where in principle $\widetilde{B} = \Gamma_0^{-1}$ (if Γ_0^{-1} exists). The $q \times q$ matrix $\widetilde{B}$ maps the reduced form dynamic shocks to the structural dynamic shocks. The objective is to identify v_{ft} and to trace out its effects on the variables of interest. If there are q common shocks, $q(q-1)/2$ restrictions are necessary to identify v_{ft} via $\widetilde{B}$.

Estimation proceeds in five steps.

Step E1: Estimate F_t from the full panel of data X_t by iterative principal components (IPC).

(i) Initialize $\delta_i(L)$ using estimates from a univariate AR($p_{x,i}$) regression in X_{it}. Let $D(L)$ be a diagonal matrix with $\delta_i(L)$ on the i-th diagonal.

(ii) Iterate until convergence

$$\min_{D(L),\Lambda,F} SSR = \sum_{t=1}^{T} \Big((I - D(L)L)X_t - \Lambda F_t\Big)'\Big((I - D(L)L)X_t - \Lambda F_t\Big).$$

(a) Let $\widehat{F}_t$ be the first k principal components of xx' using the normalization that $F'F/T = I_k$, where x is the $T \times N$ matrix of data, $F = (F_1, F_2, \ldots, F_T)'$ and k is the assumed number of static factors.

(b) Estimate $D(L)$ and Λ by regressing X_{it} on $\widehat{F}_t$ and lags of X_{it}.

The method of principal components (PC) estimates k factors as the eigenvectors corresponding to the k largest eigenvalues of $XX'/(NT)$. Under the assumption of strong factors, Bai and Ng (2006) show that the estimates are consistent for the space spanned by the true factors in the sense that $\frac{1}{T}\sum_{t=1}^{T} \left\|\widehat{F}_t - HF_t\right\|^2 = O_p(\min(N,T))$, where H is a $k \times r$ matrix of rank r. However, the idiosyncratic errors may not be white noise. Stock and Watson (2005) suggest using IPC to iteratively update $\delta_i(L)$, which is then used to define x_{it}. The static factors form the common component of x_{it}.

Step E2: Estimating the space spanned by v_{ft} Estimate a VAR in $\widehat{F}_t$ to obtain $\widehat{\Phi}_F$ and $\widehat{\varepsilon}_{Ft}$ and let $\widehat{\varepsilon}_{Xit} = x_{it} - \widehat{\Lambda}_i'\widehat{\Phi}_F\widehat{F}_{t-1}$, where $\widehat{\Lambda}$ and $\widehat{F}_{t-1}$, $\widehat{\Phi}_F$ are obtained from Step (E1). Amengual and Watson (2007) show that the q principal components of $\widehat{\varepsilon}_{Xt}$ can precisely estimate the space spanned by $\varepsilon_{ft} = \Gamma_0 v_{ft}$. An alternative is to proceed directly from the factors' VAR residuals $\widehat{\varepsilon}_{Ft}$. Bai and Ng (2007) show that q eigenvectors of $\mathrm{E}(\widehat{\varepsilon}_{Ft}\widehat{\varepsilon}_{Ft}')$ consistently estimate the space spanned by ε_{ft}, hence v_{ft}. In particular, $\frac{1}{T}\sum_{t=1}^{T}\|\widehat{\varepsilon}_{ft} - J_{NT}\varepsilon_{ft}\| = O_p(C_{NT}^{-1})$, where J_{NT} is a $q \times q$ full rank matrix and $C_{NT} = \min(\sqrt{N}, \sqrt{T})$.

Step E3: Identification of v_{ft}: The columns of common shocks $\widehat{\varepsilon}_{ft}$ are not, in general, interpretable. We seek a matrix B such that

$$\widehat{v}_{ft} = B\,\widehat{\varepsilon}_{ft}, \tag{12}$$

and $\widehat{v}_{ft}$ is a vector of mutually uncorrelated structural common shocks. The rotation matrix B differs from $\widetilde{B}$ because $\widehat{\varepsilon}_{ft}$ consistently estimates the space spanned by ε_{ft}, but not each columns separately. We consider two approaches. The first condition (abbreviated as RO) is lower triangularity of a $q \times q$ sub-matrix so that the shocks can be identified recursively from q equations. The second condition (abbreviated as BO) requires organizing the data into blocks using *a priori* information so that the factors estimated from each block can be given meaningful interpretation.

Assumption: Recursive Ordering (RO) Method (RO) is based on an assumed causal structure in short or in the long run. Just like a VAR, this would require knowledge of which of the q variables to order recursively in Y_t. For $j = 1 : q$ consider estimating the regression:

$$y_{tj} = a_{yy,j}(L)y_{t-1,j} + \sum_{k=1}^{q} a_{yf,j,k}(L)\widehat{\varepsilon}_{fkt} + v_{yt,j} \tag{13}$$

where $\widehat{\varepsilon}_{ft}$ are the q principal components of the N residuals $\widehat{e}_{Xt}$ or q eigenvectors of $\mathrm{E}(\widehat{\varepsilon}_{Ft}\widehat{\varepsilon}_{Ft}')$. Remark that we specify different autoregressive and distributed lag coefficients from FADL regression (7) since it contains $\widehat{\varepsilon}_{ft}$ and not $\widehat{v}_{ft}$. The short run zero restrictions can be imposed as follows:

i Let $\widehat{A}_{f0}$ be the estimated contemporaneous response to the q reduced-form shocks $\widehat{\varepsilon}_{ft}$:

$$\widehat{A}_{f0} = \begin{pmatrix} \widehat{a}_{yf,0,1,1} & \widehat{a}_{yf,0,1,2} & \dots & \widehat{a}_{yf,0,1,q} \\ \vdots & \vdots & \vdots & \\ \widehat{a}_{yf,0,q,1} & \widehat{a}_{yf,0,q,2} & \dots & \widehat{a}_{yf,0,q,q} \end{pmatrix}.$$

ii Define the $q \times q$ matrix $B = [\mathrm{chol}(\widehat{A}_{f0}\widehat{A}_{f0}')]^{-1}\widehat{A}_{f0}$. Now let

$$\begin{aligned} \widehat{v}_{ft} &= B\widehat{\varepsilon}_{ft} \\ \widehat{\alpha}_{yf,j} &= \widehat{a}_{yf,j}(L)B^{-1} \quad j = 1, \dots, q. \end{aligned}$$

The long run zero restrictions are imposed through similar steps. For notation purposes, let $a_{yy}(L) = \text{diag}[a_{yy,j}(L)]$, for $j = 1, \dots, q$, be the diagonal matrix polynomial of the autoregressive terms in FADL representation (13), and

$$a_{yf}(L) = \begin{pmatrix} a_{yf,1,1}(L) & a_{yf,1,2}(L) & \dots & a_{yf,1,q}(L) \\ \vdots & \vdots & \vdots & \\ a_{yf,q,1}(L) & a_{yf,q,2}(L) & \dots & a_{yf,q,q}(L) \end{pmatrix}$$

be the matrix polynomial of the distributed lag coefficients.

i. Let $\widehat{A}_{f1}$ be the estimated long run response to the q unorthogonalized shocks $\widehat{\varepsilon}_{ft}$:

$$\widehat{A}_{f1} = (I - \widehat{a}_{yy}(1))^{-1} a_{yf}(1).$$

ii. Define the $q \times q$ matrix $B = [\text{chol}(\widehat{A}_{f1}\widehat{A}'_{f1})]^{-1}\widehat{A}_{f1}$. Now let

$$\begin{aligned} \widehat{v}_{ft} &= B\widehat{\varepsilon}_{ft} \\ \widehat{\alpha}_{yf,j} &= \widehat{a}_{yf,j}(L)B^{-1}. \end{aligned}$$

By construction, $\widehat{v}_{ft}$ is orthonormal. The method achieves exact identification by using the causal ordering of the q variables selected for analysis. Hence, the identified impact (long-run) matrix on q series in y_t is lower triangular.

Imposing a causal structure through the ordering of variables is the most common way to achieve identification of FAVAR. Stock and Watson (2005) also use Assumption RO to identify the primitive shocks. Their implementation differs from ours in that we apply Choleski decomposition to the FADL estimates of $\alpha_{yf}(0)$ and hence we do not impose all the restrictions of the factor model. In contrast, Stock and Watson (2005) impose restrictions implied by the FAVAR in X_t and F_t. The results are likely to be more sensitive to the choice of X_t.

Assumption Block Ordering (BO) Method (BO) is useful when the data can be organized into blocks. Let $X = (X^1, X^2, \dots X^q)$ be data organized into q blocks. To see how data blocks facilitate identification, observe that the factor estimates $\widehat{\varepsilon}^0_{ft}$ are linear combinations of $\widehat{\varepsilon}_{Xt}$. Let $\widehat{\varepsilon}^0_f = \widehat{\varepsilon}_X \zeta^0$ be the $T \times q$ matrix of factor estimates where for each t,

$$\widehat{\varepsilon}^0_{ft} = \begin{pmatrix} \zeta^0_{11} & \zeta^0_{12} & \dots & \dots & \zeta^0_{1N} \\ \zeta^0_{21} & \zeta^0_{22} & \dots & \dots & \zeta^0_{2N} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \zeta^0_{q1} & \zeta^0_{q2} & \dots & \dots & \zeta^0_{qN} \end{pmatrix} \begin{pmatrix} \widehat{\varepsilon}_{X1t} \\ \widehat{\varepsilon}_{X2t} \\ \vdots \\ \widehat{\varepsilon}_{XNt} \end{pmatrix}. \tag{14}$$

Identification requires a priori information on the ζ.

i. For $b = 1, \dots q$, let $\widehat{\varepsilon}^b_f$ be the matrix of eigenvector corresponding to the largest eigenvalues of the $n_b \times n_b$ matrix $\widehat{\varepsilon}^{b\prime}_X \widehat{\varepsilon}^b_X$.

ii. Let B be the Choleski decomposition of the $q \times q$ sample covariance of $\widehat{\varepsilon}_{ft}$. Then $\widehat{v}_{ft} = B\widehat{\varepsilon}_{ft}$.

The identification strategy can be understood as follows. From (11), we see that $\varepsilon_{Xt} = \begin{pmatrix}\varepsilon^{1\prime}_{Xt} & \varepsilon^{2\prime}_{Xt} & \dots & \varepsilon^{q\prime}_{Xt}\end{pmatrix}'$ have ε_{ft} as common factors. Since the factors are pervasive by definition, the factors are also common to all ε^b_{Xt} for arbitrary b. Thus for each $b = 1, \dots q$, consider a factor model for $\varepsilon^b_{Xit} = \Lambda^b_i \varepsilon^b_{ft} + v^b_{Xit}$. If ε^b_{Xit} were observed, the factors for block b can be estimated by principal components which are linear combinations of series in ε^b_{Xt}. We do not observe ε^b_{Xt}, but we have $\widehat{\varepsilon}_{Xt} = x_t - \widehat{\Lambda}\widehat{\Phi}_F \widehat{F}_{t-1}$ from Step (E2). For example, if X^1 is a $T \times N_1$ panel of employment data, the first principal component of $\widehat{\varepsilon}^{1\prime}_X \widehat{\varepsilon}^1_X$ is a labor market factor $\widehat{\varepsilon}_{f1t}$, and if X^2 is a panel of price data, $\widehat{\varepsilon}_{f2t}$ is a price factor. Collecting the factors estimating from all blocks into $\widehat{\varepsilon}_{ft}$, we have

$$\widehat{\varepsilon}_{ft} = \begin{pmatrix} \zeta^1_{1,1:N_1} & 0 & 0 & \dots & 0 & 0 \\ 0 & \zeta^2_{1,1:N_2} & 0 & \dots & 0 & 0 \\ \vdots & 0 & \vdots & & \vdots & \vdots \\ 0 & 0 & 0 & \dots & 0 & \zeta^q_{1,1:N_q} \end{pmatrix} \begin{pmatrix} \widehat{\varepsilon}^1_{Xt} \\ \widehat{\varepsilon}^2_{Xt} \\ \vdots \\ \widehat{\varepsilon}^q_{Xt} \end{pmatrix} \tag{15}$$

Obviously, the factors are defined by assuming a structured covariance relation in the observables. The appeal is that we can now associate the q factors with the block of variables from which they are estimated. However, these factors can still be correlated across blocks. To orthogonalize them, step (ii) performs q regressions beginning with $\widehat{v}_{f1} = \widehat{\varepsilon}^1_f$. For $m = 2, \dots q$, $\widehat{v}_{fb} = M_b \widehat{\varepsilon}^b_f$ are the residuals from projecting $\widehat{\varepsilon}^b_f$ onto the space orthogonal to $\widehat{v}_{f1}, \dots, \widehat{v}_{f,b-1}$, and M_b is the corresponding projection matrix.

Bernanke, Boivin, and Eliasz (2005) treat the interest rate as an observed factor, organize the macro variables into a fast and a slow block, and estimate the one factor from the slow variables. Their identification is based on a Choleski decomposition of the residuals in the slow variables and the observed factor. Their implementation is specific to the question under investigation while our methodology is general. Our identification algorithm is generic, provided blocks of variables with meaningful interpretation can be defined.[4]

In conventional VAR models, the structural impulse responses are obtained by rotating the reduced form impulse response matrix by a matrix, say, B. The primitive shocks are then obtained by rotating the reduced form errors with the inverse of the same matrix. In our setup, identification of structural common shocks precedes estimation of the impulse responses. This allows us to impose additional economic restrictions on the impulse response functions without simultaneously affecting the structural shocks. We only presented Wold causal identification structures. However, medium run (à la Barsky and Sims (2011)) and sign restrictions can be imposed.

Step E4: Constructing Impulse Response Functions: Given identified $\widehat{v}_{ft}$ from the previous step, estimate a univariate FADL by OLS:

$$y_t = \alpha_{yy}(L) y_{t-1} + \alpha_{yf}(L) \widehat{v}_{ft} + v_{yt} \tag{16}$$

where $\alpha_{yy}(L)$ is a polynomial of order p_y, and α_{yf} is of order p_f. These polynomial orders can be jointly estimated by Akaike or Bayesian information criteria. The estimated responses of y_t to a

[4] Moench and Ng (2011) construct regional factors from data organized geographically. Ludvigson and Ng (2009) study the relative importance of the factor loadings and find that factor one loads heavily on real activity series, factor two on money and credit variables, while factor three loads on price variables.

unit increase in the common shocks $\widehat{v}_{ft}$ and idiosyncratic shocks v_{yt} are defined by

$$\widehat{\psi}_y^f(L) = \frac{\widehat{\alpha}_{yf}(L)}{1-\widehat{\alpha}_{yy}(L)L} \qquad \widehat{\psi}_y^y(L) = \frac{1}{1-\widehat{\alpha}_{yy}(L)L}.$$

Since $\widehat{\alpha}_{yy}(L)$ is a scalar rational polynomial, the impulse responses are easy to compute using the FILTER command in MATLAB. The least squares estimates $[\widehat{\alpha}_{yy}(L), \widehat{\alpha}_{yf}(L)]$ converge to the least squares estimates from the infeasible regression (6), $[\widehat{\alpha}^*_{yy}(L), \widehat{\alpha}^*_{yf}(L)]$. As shown in Step E2, $\widehat{\varepsilon}_{ft}$ consistently estimates the space spanned by ε_{ft}, thus regression augmented with $\widehat{\varepsilon}_{ft}$ can be treated as though ε_{ft} is observed, see Bai and Ng (2006). However, that regression would produce unorthogonalized impulse responses and a priori restrictions in form of $B = \Gamma_0^{-1} J_{NT}^{-1}$ are needed to obtain $\widehat{\psi}_y^f(L)$. The formal proof is in the Appendix.

Note that by Assumption 1, the standard deviation of all common shocks are normalized to unity. The response to a unit shock is thus the same as the response to a standard deviation shock. Note that given interpretation of $\widehat{v}_{ft}$ identified from Step E3, additional economic restrictions on the impulse responses can be directly imposed on α_{yf} for any variable y_t being in Y_t, X_t or X_t^{OTH}. For example, suppose we have a strong (*a priori*) reason that a particular (sectoral) total factor productivity (TFP) series does not react to a technology news shock even at one period lag. This implies restricting $\alpha_{yf,0,k} = \alpha_{yf,1,k} = 0$ where k reflects the news shock in $\widehat{v}_{ft}$. Hence, it restricts the response of that particular variable without affecting the identification of common shocks nor the estimation of the impulse responses of other series in the system. Moreover, one could produce impulse responses directly by adapting (16) to local projections as in Jorda (2005).

Step E5: Model Validation Our maintained assumptions are that F_t are pervasive amongst X_t rather than (X_t, X_t^{OTH}) and by assumption, X_t have a strong factor structure. We refer to X as a 'training sample'. This is useful because once the estimated common shocks $\widehat{v}_{ft}$ are available, they can be treated as regressors in a FADL model for z_t (scalar) not necessarily in X_t. This is because if (X_t, X_t^{OTH}) have a factor structure, the shocks v_{ft} common to X_t are also common to variables in X_t^{OTH}. If the common factors are important for $z_t \subset X_t^{OTH}$, then FADL coefficients on v_{ft} and its lags should be statistically significant.

4 Relation to the Other Methods and Allowing for Observed Factors

An important difference between our approach and existing structural FAVAR analysis is that we estimate the impulse responses directly rather than inverting a VAR. Chang and Sakata (2007) estimates the shocks as residuals from long vector autoregressions in observed variables. The authors show that their estimated impulse responses are asymptotically equivalent to the local projections method used in Jorda (2005). Dufour and Renault (1998) have proposed the local projections to study the Granger-causality at different horizons. Fève and Guay (2009) develops similar SVAR framework to study the effects of news shocks on worked hours. Our analysis has the additional complication that the factors are latent. Thus, we first estimate the space spanned by common factors, then estimate the space spanned by the common shocks, before finally estimating the impulse response functions.

It is useful to relate our estimate of $\Psi^f(L)$ with the conventional FAVAR approach which starts with the representation

$$\begin{pmatrix} F_t \\ X_t \end{pmatrix} = \begin{pmatrix} \Phi & 0 \\ \Lambda\Phi & D(L) \end{pmatrix} \begin{pmatrix} F_{t-1} \\ X_{t-1} \end{pmatrix} + \begin{pmatrix} \varepsilon_{Ft} \\ \Lambda\varepsilon_{Ft} + v_{Xt} \end{pmatrix}$$

from which it follows that

$$\begin{aligned} x_t &= \Lambda\Phi L(I - \Phi L)^{-1}\varepsilon_{Ft} + \Lambda\varepsilon_{Ft} + v_{Xt} \\ &= \Big(\Lambda\Phi L(I - \Phi L)^{-1} + \Lambda\Big)\varepsilon_{Ft} + v_{Xt}. \end{aligned}$$

The dynamic effects of shocks ε_{Ft} to the static factors on (prewhitened) data are determined by

$$\Lambda\Big(\Phi L(I - \Phi L)^{-1} + I\Big) = \Lambda \sum_{j=0}^{\infty} \Phi^{i+1} L^{i+1}. \tag{17}$$

At lag j, the $N \times N$ response matrix

$$\Lambda\Phi^j = \begin{pmatrix} \Lambda_i' & \Lambda_2' & \vdots & \Lambda_N' \end{pmatrix}' \Phi^j.$$

Intuitively, the total effect of ε_{Ft} depends on X_t through F_t and hence depends on the dynamics of F_t and the importance of the factor loadings on X_t. Assuming that the reduced form shocks are related to the structural shocks via $\varepsilon_{Ft} = A_0 v_{ft}$, the response to the structural shocks estimated by a FAVAR is

$$\widehat{\Psi}_j^f = \widehat{\Lambda}\widehat{\Phi}^j A_0^{-1}$$

which is a product of three terms: two that are the same for all i, and one $(\widehat{\Lambda})$ that is specific to unit i's. Since $\widehat{\Lambda}_i$ is only available for any $x_{it} \in X_t$, Ψ_j^f can be constructed only for N series. This is a consequence of the fact that the FAVAR estimates the impulses without directly estimating v_{ft}. Since we estimate v_{ft}, we can construct impulse responses for series not in X.

In contrast, our estimator of Ψ_j^f is $\widehat{\Lambda_i'\Phi^j A}_0$, which may not equal $\widehat{\Lambda}_i'\widehat{\Phi^j}A_0$, because we do not fully impose restrictions of the dynamic factor model on the static factor representation. Instead of a large FAVAR system, we estimate the FADL one variable at a time. Cross parameter restrictions between $\alpha_{yf}(L)$ and $\alpha_{yy}(L)$ are also not imposed. As is usually the case, system estimation is more efficient if the restrictions are true. However, misspecification in one equation can adversely affect the estimates of all equations. This possibility increases with N. The single equation FADL estimates are more robust to misspecification than those that rely on a large number of overidentifying restrictions which are often imposed on variables that are not of primary interest, or whose factor structure may not be strong.

Finally, restrictions on Γ_0 and Λ_0 alone may not be enough for identification. Consequently, it is not always easy to directly define A_0. FAVARs typically require several auxiliary regressions to determine A_0. In addition to incurring sampling variations at each step, the identification procedure requires tricks that are problem specific. In a FADL setting, the restrictions are directly imposed when the FADL is estimated. It is more straightforward, as will be illustrated in Section 6.

4.1 Extension to m Observed Factors

Some economic analysis involves identification of shocks to observed variables in the presence of latent shocks. For example, Bernanke, Boivin, and Eliasz (2005), Stock and Watson (2005) and Forni and Gambetti (2010) consider identification of monetary policy shocks in the presence of other shocks, using the information that some variables have instantaneous, while others have delayed response to shocks to the observed factor, being the Fed Funds Rate. These studies, summarized in Appendix B, impose restrictions of the factor models on all series. Our proposed FADL approach imposes significantly fewer restrictions on the factor model.

To extend the dynamic factor model to allow for m observed common factors w_t, let

$$\begin{aligned} X_t &= \widetilde{\lambda}^f(L)f_t + \widetilde{\lambda}^w(L)w_t + u_{Xt} \\ u_{Xt} &= D(L)u_{Xt-1} + v_{Xt} \\ \begin{pmatrix} f_t \\ w_t \end{pmatrix} &= \begin{pmatrix} \Gamma_{1,ff} & \Gamma_{1,fw} \\ \Gamma_{1,wf} & \Gamma_{1,ww} \end{pmatrix} \begin{pmatrix} f_{t-1} \\ w_{t-1} \end{pmatrix} + \begin{pmatrix} \Gamma_{0,ff} & \Gamma_{0,fw} \\ \Gamma_{0,wf} & \Gamma_{0,ww} \end{pmatrix} \begin{pmatrix} v_{ft} \\ v_{wt} \end{pmatrix} \end{aligned}$$

with $\Gamma_{1,fw} \neq 0$ and $\Gamma_{0,wf} \neq 0$. Without these assumptions, w_t is weakly exogenous and can be excluded from the analysis. The FADL representation of X_t is

$$\begin{aligned} X_t &= \begin{pmatrix} \widetilde{\lambda}^f(L) & \widetilde{\lambda}^w(L) \end{pmatrix} (I - \Gamma_1)^{-1}\Gamma_0 \begin{pmatrix} v_{ft} \\ v_{wt} \end{pmatrix} + (I - D(L)L)^{-1})v_{Xt} \\ &= \Psi^f(L)v_{ft} + \Psi^w(L)v_{wt} + \Psi^X(L)v_{Xt}. \end{aligned}$$

Let W_t be a vector consisting of w_t and its lags. Assume that its joint dynamics with F_t can be represented by a VAR(1):

$$\begin{pmatrix} F_t \\ W_t \end{pmatrix} = \begin{pmatrix} \Phi_{FF} & \Phi_{FW} \\ \Phi_{WF} & \Phi_{WW} \end{pmatrix} \begin{pmatrix} F_{t-1} \\ W_{t-1} \end{pmatrix} + \begin{pmatrix} \varepsilon_{Ft} \\ \varepsilon_{Wt} \end{pmatrix}. \tag{18}$$

The static factors are estimated from prewhitened data that also nets out the effects of the observed factors, and the construction of the structural shocks v_{ft} must take into account that the reduced form innovations to the static factors can be correlated with the innovations to the reduced form representation of the observed factors. Let $x_{it} = X_{it} - \delta_i(L)X_{it-1}$ and define

$$x_{it} = \Lambda_i^F F_t + \Lambda_i^W W_t + v_{Xit}.$$

The steps can be summarized as follows.

Step W1: Estimate F_t conditional on W_t by iterating until convergence

$$\min_{D(L),\Lambda,F} = \sum_{t=1}^{T} \Big((I - D(L)L)X_t - \Lambda^F F_t - \Lambda^W W_t \Big)' \Big((I - D(L)L)X_t - \Lambda^F F_t - \Lambda^W W_t \Big).$$

(i) Initialize $\delta_i(L)$ and Λ_i^W using estimates from a univariate AR($p_{x,i}$) regression in X_{it} augmented by W_t.

(ii) Let $\widehat{F}_t$ be the k principal components of xx' using the normalization that $F'F/T = I_k$.

(iii) Estimate Λ^F, Λ^W and $D(L)$ by regressing X_{it} on $\widehat{F}_t$, W_t and lags of X_{it}.

Step W2: Estimate Φ_F and Φ_W from a VAR in $\widehat{F}_t$ and W_t, respectively. Also let $(\widehat{\varepsilon}_{Ft}\ \widehat{\varepsilon}_{Wt})$ be the residuals from estimation of (18).

Step W3: Estimate v_{ft} and v_{wt}:

(i) Let $\widehat{\varepsilon}_{Xit} = x_{it} - \widehat{\Lambda}'_{iF}\widehat{\Phi}_F\widehat{F}_{t-1} - \widehat{\Lambda}'_{iW}\widehat{\Phi}_W W_{t-1}$, where $\widehat{F}_t$ are the iterative principal components of the full panel. Estimate reduced-form common shocks, $\widehat{\varepsilon}_t = (\widehat{\varepsilon}'_{ft}\ \widehat{\varepsilon}'_{wt})'$, as principal components of $\widehat{\varepsilon}_{Xit}$, or as eigenvectors from covariance matrix of VAR residuals $(\widehat{\varepsilon}_{Ft}\ \widehat{\varepsilon}_{Wt})$.

(ii) Let $X = (X^1, X^2, \ldots X^q)$ be data organized into q blocks. For $b = 1, \ldots q$, let $\widehat{\varepsilon}_{fb}$ be the eigenvector corresponding to largest eigenvalue of the $n_b \times n_b$ matrix $\widehat{\varepsilon}'_{X^b}\widehat{\varepsilon}_{X^b}$.

(iii) Orthogonalize $\widehat{\varepsilon}_t$ using the causal or block ordering of the variables to get $\widehat{v}_{ft}$ and $\widehat{v}_{wt}$.

Step W4: Construct the impulse responses: Estimate by least squares (with restrictions on $\alpha_{yf}(L)$ and $\alpha_{yw}(L)$):

$$y_t = \alpha_{yy}y_{t-1} + \alpha_{yf}(L)\widehat{v}_{ft} + \alpha_{yw}(L)\widehat{v}_{wt} + v_{yt}. \tag{19}$$

Then $\widehat{\psi}^f(L) = \frac{\widehat{\alpha}_{yf}(L)}{(1-\widehat{\alpha}_{yy}(L))}$ gives the response of y_t to v_{ft} holding W_t fixed, when the latter is considered as a control. In addition, $\widehat{\psi}^w(L) = \frac{\widehat{\alpha}_{yw}(L)}{(1-\widehat{\alpha}_{yy}(L))}$ gives the response of y_t to v_{wt}.

5 Simulations

We use two sets of simulation exercises to evaluate the finite sample properties of the identified impulse responses. In the first case, the data generating process consists of the DFM (1)-(3) , while in the second experiment the data are simulated from a multi-sector dynamic stochastic general equilibrium (DSGE) model following Ruge-Murcia and Onatski (2010).

5.1 Experiment I

Data are simulated from equations (1)-(3) with $\widetilde{\lambda}(L)$ being a polynomial of degree 1. The persistence parameter δ_i is uniformly distributed over (.2,.5). The errors v_{Xit}, v_{ft} and the non-zero factor loadings are normally distributed with variances $\sigma^2_X, 1, \sigma^2_\lambda$ respectively. We set $T = 200$ and $N = 120$ to mimic the macroeconomic panels used in empirical work.

The structural moving-average representation is

$$X_{it} = \begin{pmatrix}\widetilde{\lambda}_{0i} & \widetilde{\lambda}_{1i}L\end{pmatrix}\begin{pmatrix}I - \Gamma_1 L\end{pmatrix}^{-1}\Gamma_0\begin{pmatrix}v_{f1t}\\ v_{f2t}\end{pmatrix} + (1-\delta_i L)^{-1}v_{Xit}.$$

This implies that the impact response of X_{it} to the shocks is summarized by

$$X_{it} = (\widetilde{\lambda}_{0,i,1} \quad \widetilde{\lambda}_{0,i,2})\begin{pmatrix}\gamma_{0,11} & \gamma_{0,12}\\ \gamma_{0,21} & \gamma_{0,22}\end{pmatrix}\begin{pmatrix}v_{f1t}\\ v_{f2t}\end{pmatrix} + v_{Xit}. \tag{20}$$

DGP 1: $q = 2$ factors, $\Gamma_1 = \begin{pmatrix}0.75 & 0\\ 0 & 0.7\end{pmatrix}$, $\sigma_{\lambda,1k} = 1$.

case a: $\Gamma_0 = I$, **case b:** $\Gamma_0 = \begin{pmatrix} 1 & 0 \\ 0.5 & 1 \end{pmatrix}$, $\sigma_{\lambda,2k} = 0.8$.

The N variables are ordered such that the first $N/2$ variables respond contemporaneously to both shocks and are labeled 'fast'. The last $N/2$ do not respond contemporaneously to shock 2 and are labeled slow'. By design, X_{1t} is a fast variable and X_{Nt} is a slow variable. This structure is achieved by specifying

$$(\widetilde{\lambda}_{0,i,1} \quad \widetilde{\lambda}_{0,i,2}),\ \ i = 1, \ldots, N/2 \qquad \text{and} \qquad (\widetilde{\lambda}_{0,i,1} \quad 0) \ \ i = N/2+1, \ldots, N.$$

Let $Y_t = (X_{1t}, X_{N,t})$ be the two variables whose impulse responses are of interest. Since there are no observed factors, estimation begins with E1 and E2. We consider both identification strategies and estimate two FADL regressions, one for each variable in Y_t. As a benchmark, we also estimate the (infeasible) FADL regressions using the true common shocks, v_{ft}.

The results are summarized in Table 1. The top panel shows that for DGP 1a, the correlation between v_{fjt} and $\widehat{v}_{fjt}$ are well above 0.90 for both identification strategies. For DGP 1b, Method (a) is more precise than (b) but the latter is still quite precise. The correlation between v_{fjt} and v_{fkt}, $i \neq k$, are numerically small. The middle panel of Table 1 reports the RMSE of the estimated impulse responses when the shocks are observed. Given that there are two shocks, there are two impulse responses to consider for each of the two variables. We use $v_{fj} \to X_k$ to denote the response of X_k to shock j, where $k = 1$ is the fast variable, and $k = N$ is the slow variable. The third panel reports results when the common shocks have to be estimated. The $\widehat{\psi}$ are practically identical to the analytical ones given by (20). Furthermore, the impact response of slow variable to second shock is not statistically different from zero (not reported).

DGP 2: $q = 2$ latent and $m = 1$ observed factors Let $\sigma_{\lambda,jk} = 1$, $\sigma_{\lambda,1k} = 1$, $\sigma_{\lambda,2k} = 0.8$, and $\sigma_{\lambda,3k} = 0.7$ and $\Gamma_1 = \begin{pmatrix} 0.75 & 0 & 0 \\ 0 & 0.7 & 0 \\ 0 & 0 & 0.65 \end{pmatrix}$.

case a: $\Gamma_0 = I$, **case b:** $\Gamma_0 = \begin{pmatrix} 1 & 0 & 0 \\ 0.4 & 1 & 0 \\ 0.3 & 0.2 & 1 \end{pmatrix}$.

The ordering of structural shocks is $v_{ft} = (v_{ft}^{slow}, \ \ v_{ft}^{mp}, \ \ v_{ft}^{fast})$. The goal is (partial) identification of the effects of v_{ft}^{mp}. Again, the N variables are divided between fast and slow: slow variables do not respond on impact to second and third shocks, and at least one variable does not respond immediately only to the third shock, such that the causal ordering holds.

After the common shocks are estimated and identified according to Methods RO and BO, two FADL regressions are estimated for the two components in Y_t: one fast and one slow variable. The results are summarized in Table 2. As we are interested in partial identification of the second shock, we only report results on the approximation of v_{ft}^{mp}, and impulse responses of two variables to this shock. As in the previous exercise, FADL regressions with true shocks produce impulse response coefficients practically identical to the analytical ones. The estimated second shock is very close to the true one and $\widehat{\psi}(L)$ very close to true impulse response coefficients.

5.2 Experiment II

The previous simulation exercise served to evaluate if the estimation of FADL representation is able to recover the true shocks and generate accurate impulse responses when the DGP is a DFM. Now, we replicate exactly the same simulation design from Ruge-Murcia and Onatski (2010): a large multi-sector DSGE able to produce hundreds of aggregate and disaggregate series. The objective is to use this laboratory to evaluate if the FADL methodology can recover the impulse responses to an exogenous monetary policy shock and compare its performance to standard FAVAR analysis.

We proceed in following steps:

1. Simulate 156 series that consist of: 6 aggregate and 150 sectoral variables (output, hours worked, wages, consumption and inflation). All the details are in Ruge-Murcia and Onatski (2010) and Matlab codes are publicly available.

2. Estimate FADL and FAVAR models on simulated data and produce impulse responses to exogenous money growth shock. Compare their performance in terms of mean (and median) squared (and absolute) errors.

The previous steps are repeated 1000 times. The results for aggregate series are summarized in Table 3. For the sake of space, additional results and estimation details are presented in the not-for-publication online Appendix. Overall, FADL models provide better approximations of true impulse responses of aggregate series, especially when the performance is measured by median RMSE and by mean absolute error. Also, we remark that estimating common shocks from a subset of 118 series, out of 156, often shrinks the estimation error.[5] Moreover, the uncertainty around median impulse response estimates is much smaller in case of FADL model. Similar results for sectoral series are summarized in the online Appendix.

6 Two Examples

In this section, we use FADL to analyze two problems: measuring the dynamic effects of monetary policy shocks and news (technology) shocks.

6.1 Example 1: Effects of a Monetary Policy Shock

As in Bernanke, Boivin, and Eliasz (2005), the monetary authority observes N_{slow} variables (such as measures of real activity and prices) collected into X_t^{slow} when setting the interest rate R_t but does not observe N_{fast} variables (such as financial data) collected into X_t^{fast}. In this exercise, R_t is an observed factor. Let $v_{ft} = (v_{ft}^{slow}, \quad v_{ft}^{mp}, \quad v_{ft}^{fast})$ be the vector of q common shocks, where v_{ft}^{mp} is the monetary policy shock, v_{ft}^{fast} is a vector q_1 shocks, specific to X^{fast}, and v_{ft}^{slow} is a vector of q_2 shocks, specific to X_t^{slow} respectively, with $q = q_1 + q_2 + 1$. The issue of interest is (partial) identification of the effects of monetary policy shock, meaning that the effects due to v_{ft}^{slow} and v_{ft}^{fast} are not of interest.

[5]A set of sectoral series, that do not respond significantly enough to money growth shock, have been removed from X_t and form the dataset X^{OTH}. See tables in the online Appendix.

Bernanke, Boivin, and Eliasz (2005) identify the monetary policy shock by assuming that Ψ_0^f is a block lower triangular structure. This involves restrictions on $N_{slow} > q_2$ variables. In a data rich environment, some of these restrictions could well be invalid. Instead, we consider an alternative identification strategy using fewer restrictions. It is based on Assumption RO which can be achieved by choosing the first q variables to compose of q_1 (slow) indicators of real activity and prices, followed by the monetary policy instrument.[6] The Assumption BO, which identifies the shocks at the block level, is also easy to apply. The data are ordered as $Y_t = (X_t^{slow\prime}, \quad R_t, \quad X_t^{fast\prime})'$. After estimating v_{ft}^{slow} from X_t^{slow} and and v_{ft}^{fast} from X_t^{fast}, the monetary policy shocks are the residuals from a regression of R_t on current and lag values of $\widehat{v}_{ft}^{slow}$. By construction, the estimated structural shocks are mutually uncorrelated under both RO and BO assumptions. A FADL in all the shocks is then estimated for each variable of interest.

In terms of matrix Ψ_0^f, Bernanke, Boivin, and Eliasz (2005) assumes:

$$\Psi_0^f = \begin{pmatrix} \underbrace{\psi_{0,1,1}}_{N_{slow}\times q_1} & \underbrace{0}_{N_{slow}\times 1} & \underbrace{0}_{N_{slow}\times q_2} \\ \underbrace{\psi_{0,2,1}}_{1\times q_1} & \underbrace{\psi_{0,2,2}}_{1\times 1} & \underbrace{0}_{1\times q_2} \\ \underbrace{\psi_{0,3,1}}_{N_{fast}\times q_1} & \underbrace{\psi_{0,3,2}}_{N_{fast}\times 1} & \underbrace{\psi_{0,3,3}}_{N_{fast}\times q_2} \end{pmatrix}.$$

Assumptions RO and BO both assume that the top $q \times q$ block of Ψ_0^f is lower triangular:

$$\Psi_{0,1:q,1:q}^f = \begin{pmatrix} \underbrace{\psi_{0,1:q,1}}_{q_1\times q_1} & \underbrace{0}_{q_1\times 1} & \underbrace{0}_{q_1\times q_2} \\ \underbrace{\psi_{0,2:q,1}}_{1\times q_1} & \underbrace{\psi_{0,2:q,2}}_{1\times 1} & \underbrace{0}_{1\times q_2} \\ \underbrace{\psi_{0,3:q,1}}_{q_2\times q_1} & \underbrace{\psi_{0,3:q,2}}_{q_2\times 1} & \underbrace{\psi_{0,3:q,3}}_{q_2\times q_2} \end{pmatrix}.$$

However, the Ψ_0^f matrix and $\widehat{v}_{ft}$ identified by RO will be different from those identified by BO. Under Assumption RO, all N series are used to estimate the q vector ε_{ft}. Thus any q series in the training sample can be used to identify primitive shocks v. Under Assumption BO, ε_{ft}^j is estimated from block j of X_t. Thus, the j shock in v_{ft} is identified from one of the N_j series in block j of X_t. Assumption BO also allows *a priori* economic restrictions to be imposed on some or all variables within the blocks. For example, we can restrict all N_{slow} series not to react on impact to a monetary policy shock, while the response of fast moving variables is unrestricted. Since these restrictions are imposed on equation by equation basis, they do not affect the estimation nor the identification of structural shocks.[7]

[6] One may also add q_2 financial indicators at the end of the recursion, but Bernanke, Boivin, and Eliasz (2005) found that there is little informational content in the fast moving factors that is not already accounted for by the federal funds rate.

[7] The restrictions can vary across series in the block. For example, one series could be restricted to respond only 2 periods after the shock, the sign of another variables could be fixed, the shape of the impulse response function could be constrained for a third variables, and so on.

6.1.1 Data and Results

The training sample used to estimate the factors consists of 107 quarterly aggregate macroeconomic and financial indicators over the extended sample 1959Q1- 2009 Q1. This data set consists of fast and slow moving variables. The Federal funds rate (FFR) is treated as an observed factor. All data are assumed stationary or transformed to be covariance stationary. The complete list of variables is given in the online Appendix.

Our estimation differs from Bernanke, Boivin, and Eliasz (2005) in two ways. First, we use quarterly data. Second, we estimate the factors by IPC to take care of autocorrelation in residuals. According to information criteria in Amengual and Watson (2007) and Bai and Ng (2007), there are $q = 3$ latent dynamic factors in the training sample. Identification is achieved by imposing a causal ordering. We order commodity price inflation first, followed by GDP deflator inflation, unemployment rate, and then FFR. Hence monetary policy is the last variable in this causal ordering, which implies zero contemporaneous response to monetary policy by the slow moving variables. We only impose restrictions on q series (one from each block) while Bernanke, Boivin, and Eliasz (2005) impose restrictions on all series belonging to the slow moving block.

Compared to Stock and Watson (2005), we impose the same minimal number of restrictions to identify the structural shocks, but our approach differs in estimating the impulse response functions. Instead of constructing impulse response coefficients of X_t as $(I-\widehat{D}(L))\widehat{\Lambda}(I-\widehat{\Gamma}_1(L))^{-1}\widehat{\Gamma}_0$, we rather estimate the product, $\psi_i^f(L)$, equation by equation for any element of X_t and X_t^{OTH}.

The 12 period impulse responses are presented in Figure 1. As in Bernanke, Boivin, and Eliasz (2005), controlling for the presence of common shocks resolves anomalies found in the literature. After a monetary policy shock, the fast moving variables such as Treasury bills increase immediately, while stock prices, housing starts, and consumer expectations fall. Furthermore, many measures of the slow variables including real activity and prices decline as a result of the shock without evidence of a price puzzle. The exchange rate appreciates fully on impact, with no evidence of overshooting. The results for the variables of interest are in line with Christiano, Eichenbaum, and Evans (2000) who use recursive and non-recursive identification schemes to study the effects of monetary policy, using small VARs. However, once the common shocks are estimated, the effects of monetary policy can be studied for many variables, not just the q variables used in identification. The scope of the analysis is much larger than a small VAR.

To check the validity of the factor structure in series not in the training sample, we consider X_t^{OTH} consisting of hundreds of disaggregated series. Amongst these are (i) sectoral CPI, PCE, and PPI measures of inflation, (ii) disaggregated employment series, (iii) investment measures (iv) 18 consumption series, and (v) industrial production sectoral data. For each of these additional variables, the Wald test is used to test the null hypothesis that all coefficients in $\alpha_{yf}(L)$ are jointly zero. The null hypothesis cannot be rejected at the five percent level for many series including one sectoral CPI, 15 PCE, one employment, one investment and two consumption series. For these series, the data does not support the presence of a factor structure.

We then proceed to analyze the effects of monetary policy on variables in X_t^{OTH}. Interestingly, the impulse responses of variables not affected by v_{ft} display price-puzzle like features. As seen in the top panel of Figure 2 for some of these variables, an increase in the Fed Funds rate increases rather than lowers prices. The bottom panel displays results for four series with significant $\widehat{\alpha}_{yf}(L)$. For these latter set of variables, the impulse responses are similar to those reported for the variables in the training sample, namely, that an increase in the Fed funds rate lowers prices.[8]

6.1.2 Asymmetric Effects of a Monetary Policy Shock

In order to show the flexibility of the FADL approach, we now verify the asymmetry of monetary policy effects. In the standard VAR (and FAVAR) approach, one must specify and estimate a nonlinear model to allow for asymmetric impulse responses.[9]. In FADL framework this is easily handled by adding positive and negative values of the identified monetary policy shock time series separately to each univariate regression in equation (16). Let $\widehat{v}_{ft}^{MP+}$ be a time series of positive monetary policy shocks and zero elsewhere, and $\widehat{v}_{ft}^{MP-}$ be a vector containing negative shocks. We remove $\widehat{v}_{ft}^{MP}$ from FADL regression 16 and add $[\widehat{v}_{ft}^{MP+}\ \widehat{v}_{ft}^{MP-}]$. Then, impulse responses to both contractionary and expansionary monetary policy shocks are easily computed by inverting the autoregressive part. Another approach is adopted by Angrist, Jorda, and Kuersteiner (2013) and Barnichon and Matthes (2014) who approximate the impulse responses directly from the MA representation using Gaussian basis functions and with a semi-parametric estimator based on propensity score weighting respectively. Our framework is computationally much easier, is not subject to non-fundamentalness problem and allows to study the asymmetry of monetary policy effects on hundreds of economic and financial indicators.

The results are presented in Figures 3 to 6. We find that contractionary monetary policy causes more significant effects on several measures of economic activity. First, Figure 3 shows the impulse responses of GDP, Industrial production (IP), Employment and Consumption after contractionary and expansionary monetary policy shocks. It reveals that the response of these real activity variables are much less significant after the expansionary shock compared to their responses to a contractionary policy, suggesting that only (unexpected) tightening in monetary policy have important (statistical and economical) impacts. The IRFs of all 24 series are presented in online Appendix. Second, the comparison of point estimates in Figure 4 shows that, in general, the asymmetric contractionary shock produces more pronounced effects on real activity, consumption, credit and housing starts than in the case of the average impulse. Third, expansionary shock generates lot of movements in these series compared to the symmetric responses, see Figure 5, except for the consumer expectations that reacts much more. Fourth, we find quite symmetric effects of the monetary policy on financial markets, as measured by the IRFs of Treasury Bills and SP500 returns. Finally, Figure 6 shows the variance decomposition after the asymmetric contractionary and expansionary monetary policy shocks. Interestingly, the expansionary shock is much more important for interest rate, M2 and consumer expectations, while the contractionary shock dominates for real activity series.

[8] The impulse responses of all sectoral variables from X_t^{OTH} are presented in the online Appendix. The responses of many disaggregated series are in line with theory: a decline of real activity and price indicators across several sectors after an adverse monetary policy shock. In case of employment variables, only mining and government sector series diverge from others during the first year after the shock, while the price indicators of some nondurable goods sectors present the price puzzle behavior.

[9] See Cover (1992) and Weise (1999) among others.

6.2 Example 2: Effects of a News Shock

Beaudry and Portier (2006) consider technology and news shocks, $v_{ft} = (v_t^{TFP}\ v_t^{NS})'$, interpreted as an announcement of future change in productivity. They are interested in the effects of these two shocks on productivity X_{1t}. Consider identification by the short run restrictions. Suppose that the first N_1 variables $X_t^1 \subset X_t$ do not respond immediately to v_t^{NS}, but its response to v_t^{TFP} is unrestricted. Then Ψ_0^f is lower block triangular, viz:

$$\Psi_0^f = \begin{pmatrix} \psi_{0,1,1}^f & 0 \\ \vdots & \vdots \\ \psi_{0,N_1+1,1}^f & \psi_{0,N_1+1,2}^f \\ \vdots & \vdots \\ \psi_{0,N,1}^f & \psi_{0,N,2}^f \end{pmatrix} \equiv \begin{pmatrix} \Psi_{0,1:N_1,1}^f & 0_{1:N_1,1} \\ \cdots & \cdots \\ \Psi_{0,N_1+1:N,1}^f & \Psi_{0,N_1+1:N,2}^f \end{pmatrix}.$$

This structure can be achieved if Λ_0 and Γ_0 are both *lower* block triangular, ie.

$$\underbrace{\Lambda_0}_{N\times 2} = \begin{pmatrix} \lambda_{0,1,1} & 0 \\ \vdots & \vdots \\ \lambda_{0,N_1+1,1} & \lambda_{0,N_1+1,2} \\ \vdots & \vdots \\ \lambda_{0,N,1} & \lambda_{0,N,2} \end{pmatrix} = \begin{pmatrix} \Lambda_{0,1:N_1,1} & \vdots & 0_{N_1\times 1} \\ \Lambda_{0,N_1+1:N,1} & \vdots & \Lambda_{0,N_1+1:N,2} \end{pmatrix} \quad \text{and} \quad \underbrace{\Gamma_0}_{2\times 2} = \begin{pmatrix} \Gamma_{0,11} & 0 \\ \Gamma_{0,21} & \Gamma_{0,22}. \end{pmatrix}$$

The zero restriction should hold for all series in the first block. But since there are only two shocks, any two series permit exact identification provided one is from X_t^1, one from X_t^2, and one restriction is imposed on Ψ_0^f. Beaudry and Portier (2006) only uses two variables (X_{1t}, X_{Nt}) for analysis where X_{1t} is a measure of TFP and X_{Nt} is stock price. We allow for $N > 2$ variables. But unlike standard VARs which require restrictions of order N^2 to identify N shocks, we use q series to exactly identify $q = 2$ shocks. As discussed earlier, instead of putting restrictions on Γ_0 or Λ_0 separately, our restrictions are imposed on the relevant row(s) of $\Psi_0^f = \Gamma_0\Lambda_0$. The bivariate system has the property that

$$\begin{pmatrix} X_{1t} \\ X_{Nt} \end{pmatrix} = \begin{pmatrix} \psi_{0,11}^f & 0 \\ \psi_{0,21}^f & \psi_{0,22}^f \end{pmatrix} \begin{pmatrix} v_t^{TFP} \\ v_t^{NS} \end{pmatrix} + \sum_{j=1}^{\infty} \begin{pmatrix} \psi_{j,11}^f & \psi_{j,12}^f \\ \psi_{j,21}^f & \psi_{j,22}^f \end{pmatrix} \begin{pmatrix} v_{t-j}^{TFP} \\ v_{t-j}^{NS} \end{pmatrix}.$$

The number of identifying restrictions used in the FADL is of order q^2 irrespective of N. This also contrasts with standard FAVARs which impose many overidentifying restrictions. In our setup, a large N is desirable for FADL because it improves estimation of v_{ft}. Long run restrictions can similarly be imposed so that $\Psi^f(1)$ is block lower triangular. A FADL leads to exact identification using the salient features of the factor model.

6.2.1 Data and Results

Our data consists of $X_t = (X_t^{TFP}, X_t^{SP}, X_t^{OTH})$, where X_t^{TFP} contains six TFP measures from FRB San Francisco, X_t^{SP} is a vector of eight S&P and Dow Jones aggregate stock price indicators, and X_t^{OTH} is a vector of 104 macroeconomic time series used in the previous example but with the stock prices removed.[10] Beaudry and Portier (2006) only use one series of the six series in X_t^{TFP} and one series in X_t^{SP} at the time. Forni, Gambetti, and Sala (2011) use the same TFP series and some of our stock price measures.

Two identification strategies are considered:

i (Recursive Ordering) estimate two common shocks from $X_t = (X_t^{TFP}, X_t^{SP})$. Two series, one from X_t^{TFP} and one from X_t^{SP} are selected. By ordering the TFP series first, the rotation matrix B that identifies the technology and the news shock is easily computed.

ii (Block Ordering) ε_t^{TFP} is estimated exclusively from X_t^{TFP} and ε_t^{SP} is estimated from X_t^{SP}. The identification is based on the structure

$$\begin{pmatrix} \varepsilon_t^{TFP} \\ \varepsilon_t^{SP} \end{pmatrix} = \begin{pmatrix} a_{11} & 0 \\ a_{21} & a_{22} \end{pmatrix} \begin{pmatrix} v_t^{TFP} \\ v_t^{NS} \end{pmatrix}.$$

Effectively, $\widehat{v}_t^{TFP} = \varepsilon_t^{TFP}$ and $\widehat{v}_t^{NS}$ are the residuals from a projection of $\widehat{\varepsilon}_t^{SP}$ onto $\widehat{v}_t^{TFP}$. Note that under both identification strategies the estimated shocks are mutually uncorrelated.

Once $\widehat{v}_t^{TFP}$ and $\widehat{v}_t^{NS}$ are available, variable by variable FADL equations are estimated for all series in X_t. The zero impact restrictions are imposed for all TFP measures, while all other FADL regressions are left unrestricted. The results for the two identification strategies and for news shocks v_t^{NS} are given in Figures 7 and 8. We report results for stationary (transformed) data.[11] The Table 5 contains p-values for Wald test for the null hypothesis of no factor structure in X_t^{TFP}, X_t^{SP} and X_t^{OTH} variables. The abbreviations 'RO', 'BO' stand for Assumption RO and BO respectively. The null hypothesis is strongly rejected for many series.

Of special interest here are the responses to a positive news shock. The forward looking variables such as stock prices, housing starts, new orders and consumer expectations increase on impact. Consumption reacts positively. The wealth effect does not seem important enough such that the worked hours also increase on impact.

Our results are in line with Beaudry and Portier (2006) for the pro-cyclical response of worked hours. However, Barsky and Sims (2011) also estimate positive response of consumption and find an immediate decrease of hours. Forni, Gambetti, and Sala (2011) find that both consumption and hours respond negatively on impact. These differences can be due to the choice of variables used to identify the shocks and to the variables selected for analysis. In particular, these studies used a small set of worked hours measures. We check the sensitivity of our results to a much broader set of available indicators.

To this end, we assess the sensitivity of our results (under the assumption of a block structure) to additional variables as follows:

[10]The complete list of additional variables used in news shock application is available in online Appendix.

[11]For the sake of space, the impulse responses to a positive technology shock v_t^{TFP} are reported in the online Appendix. The estimated dynamic effects of technology shocks are in line with Christiano, Eichenbaum, and Vigfusson (2003) who suggest that technology improvements are pro-cyclical for real activity and hours measure, but contrary to Basu, Fernald, and Kimball (2006) and Gali (1999).

a Estimate ε_t^{OTH} from the macro data X_t^{OTH}. Identification is now based on

$$\begin{pmatrix} \varepsilon_t^{TFP} \\ \varepsilon_t^{SP} \\ \varepsilon_t^{OTH} \end{pmatrix} = \begin{pmatrix} a_{11} & 0 & 0 \\ a_{21} & a_{22} & 0 \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{pmatrix} v_t^{TFP} \\ v_t^{NS} \\ v_t^{OTH} \end{pmatrix}.$$

b change the ordering to ε_t^{TFP}, ε_t^{OTH} with ε_t^{SP} ordered last in view of the forward looking nature of stock prices.

These results are denoted Block 2 and Block 3 respectively and correspond to $BO2$ and $BO3$ in Tables and Figures. In a VAR setup, there would be 104 VARs to consider when there are 104 macro variables that might not be econometrically exogenous to TFP and stock prices. In the factor setup, we only need to estimate one set of macro shocks from 104 macro series. As shown in Figure 8, the effects of news shocks are smaller when the macro shocks are present. In other words, omitted variables from the VAR could have biased the estimated effects of news shocks. However, for an assumed q, the identified impulse responses are robust to the ordering of the variables.

As is well known, VARs involving hours worked are sensitive to whether the hours series is in level or in difference, see for example, Fève and Guay (2009). We use the most conservative specification Block 3 to further understand the dynamic response the level and growth of average weekly hours (AWH) to a positive news shock. The results are presented in Figure 9. The dynamic responses of AWH and total hours *indices* are plotted in Figure 10. Regardless of the data transformation, the hours series are pro-cyclical after the news shock. This exercise illustrates the FADL can be used to check the robustness of the results to many other measures without affecting the identification of structural shocks.

7 Conclusion

In this paper, we have proposed a new approach to analyze the dynamic effect of common shocks in a data-rich environment. After estimating the common shocks from a large panel of data and imposing a minimal set of identification restrictions, the impulse responses are obtained from an autoregression in each variable of interest, augmented with a distributed lag of structural shocks.

The FADL framework presents several advantages. The method is more robust to a fully structural factor model when the identifying factor restrictions do not hold universally. Since the impulse responses are obtained from a set of regressions, the restrictions are easy to impose, and implications of the factor model can be tested. The estimation of common shocks is less likely to be affected by the presence of weak factors. The FADL methodology is used to measure the effects of monetary policy shocks, and to news and technology shocks. The approach allows us to go beyond existing structural FAVAR, and to validate restrictions of the factor model.

Appendix A: Consistency of Dynamic Multipliers Least Squares Estimates

Start from the infeasible regression (6) and use $v_{ft} = \Gamma_0^{-1}\varepsilon_{ft}$, $\varepsilon_{ft} = J_{NT}^{-1}\widehat{\varepsilon}_{ft}$ to get

$$\begin{aligned} y_t &= \alpha_{yy}^*(L)y_{t-1} + \alpha_{yf}^*(L)\Gamma_0^{-1}\varepsilon_{ft} + \alpha_{yf}^*(L)\Gamma_0^{-1}J_{NT}^{-1}\widehat{\varepsilon}_{ft} - \alpha_{yf}^*(L)\Gamma_0^{-1}J_{NT}^{-1}\varepsilon_{ft} + v_{yt}^* \\ &= \alpha_{yy}^*(L)y_{t-1} + \alpha_{yf}(L)\widehat{\varepsilon}_{ft} + v_{yt}, \end{aligned}$$

where $\alpha_{yf}(L) = \alpha_{yf}^*(L)\Gamma_0^{-1}J_{NT}^{-1}$ and $v_{yt} = v_{yt}^* - \alpha_{yf}(L)(\widehat{\varepsilon}_{ft} - J_{NT}\varepsilon_{ft})$. Without loss of generality, assume $p_y = 1$ and $p_f = 0$ and write the model in matrix form

$$y = \widehat{z}\delta + e, \tag{21}$$

where $\widehat{z} = (z_{yt}, \widehat{\varepsilon}_{ft}) = (y_{t-1}, \widehat{\varepsilon}_{ft})$, and $\delta = [\alpha_{yy}^*(1), \alpha_{yf}(0)]\prime$. Let $z = (y_{t-1}, J_{NT}\varepsilon_{ft})$ so that $\delta^* = [\alpha_{yy}^*(1), \alpha_{yf}^*(0)\Gamma_0^{-1}J_{NT}^{-1}]\prime$ are the parameters from infeasible FADL regression when ε_{ft} are observed. Let $S_{\widehat{z}\widehat{z}} = T^{-1}\sum_{t=1}^T \widehat{z}_t\widehat{z}_t'$. Least squares estimation of (21) yields

$$\begin{aligned} \sqrt{T}(\widehat{\delta} - \delta) &= S_{\widehat{z}\widehat{z}}^{-1}T^{-1/2}\widehat{z}'e \\ &= S_{\widehat{z}\widehat{z}}^{-1}T^{-1/2}\widehat{z}'e + S_{\widehat{z}\widehat{z}}^{-1}T^{-1/2}\widehat{z}'(\varepsilon_f J_{NT}' - \widehat{\varepsilon}_f)\alpha_{yf}(0)'. \end{aligned}$$

Following Bai and Ng (2006), $\frac{1}{T}\sum_{t=1}^T(\widehat{\varepsilon}_{ft} - J_{NT}\varepsilon_{ft})\widehat{z}_t' = O_p(C_{NT}^{-2})$, and thus

$$T^{-1/2}\widehat{z}'(\widehat{\varepsilon}_f - \varepsilon_f J_{NT}') = O_p(T^{1/2}C_{NT}^{-2}) \to 0 \quad \text{if} \quad \sqrt{T}/N \to 0.$$

In addition,

$$\begin{aligned} T^{-1/2}\widehat{z}'e &= T^{-1/2}(z_y'e \;\; J_{NT}\varepsilon_f'e) + o_p(1) \\ &= T^{-1/2}M_{NT}z'e + o_p(1), \end{aligned}$$

where $M_{NT} = \text{diag}(I, J_{NT})$ and $z' = (z_y' \;\; \varepsilon_f')$. Under standard assumptions, $T^{-1/2}z'e \to N(0, \Sigma_{ze})$, where $\Sigma_{ze} = \text{plim}S_{\widehat{z}e} = T^{-1}\sum_{t=1}^T \widehat{z}_t\widehat{z}_t'e_t^2$. Thus,

$$\sqrt{T}(\widehat{\delta} - \delta) = S_{\widehat{z}\widehat{z}}^{-1}M_{NT}T^{-1/2}z'e + o_p(1) \to N(0, \Sigma_{\widehat{\delta}}),$$

where $\Sigma_{\widehat{\delta}} = \text{plim}S_{\widehat{z}\widehat{z}}^{-1}M_{NT}\Sigma_{\widehat{z}e}M_{NT}'S_{\widehat{z}\widehat{z}}^{-1\prime}$. In addition,

$$S_{\widehat{z}\widehat{z}} = M_{NT}S_{zz}M_{NT}' + o_p(1) \to M\Sigma_{zz}M'.$$

Thus, the asymptotic covariance matrix of $\widehat{\delta}$ reduces to

$$\Sigma_{\widehat{\delta}} = M^{-1\prime}\Sigma_{zz}^{-1}\Sigma_{ze}\Sigma_{zz}^{-1}M^{-1}$$

which can be estimated by $\Sigma_{\widehat{\delta}} = S_{\widehat{z}\widehat{z}}^{-1}\Sigma_{\widehat{z}\widehat{e}}S_{\widehat{z}\widehat{z}}^{-1}$. If $\sqrt{T}/N \to c > 0$, then $T^{-1/2}\widehat{z}'(\widehat{\varepsilon}_f - \varepsilon_f J_{NT}')$ does not vanish and $\widehat{\delta}$ will have an asymptotic bias, see Ludvigson and Ng (2009) for details.

We have shown that the least squares estimates of $\alpha_{yf}(L)$ will consistently estimate $\alpha_{yf}^*(L)\Gamma_0^{-1}J_{NT}^{-1}$. Hence, to get consistent estimates of the orthogonalized impulse responses we need to impose a priori restrictions in terms of $B = \Gamma_0^{-1}J_{NT}^{-1}$.

Appendix B: Relation to Other Methods with Observable Factors

The estimated common shocks are treated as regressors of a FADL. As such, *a priori* restrictions on the impulse response functions can be directly imposed in estimation of the FADL by least squares. The approach is simpler and more transparent than existing implementations of structural FAVARs.

Consider the identification of monetary policy shocks in the presence of other shocks as in Bernanke, Boivin, and Eliasz (2005). Their point of departure is a static factor model with latent and observed factors:

$$X_t = \Lambda^F F_t + \Lambda^R R_t + u_t \tag{22}$$

$$\begin{bmatrix} F_t \\ R_t \end{bmatrix} = \Phi \begin{bmatrix} F_{t-1} \\ R_{t-1} \end{bmatrix} + \eta_t \tag{23}$$

where F_t is vector of r latent factors and R_t is the observed factor (usually Federal Funds Rate or 3-month Treasury Bill). The authors organize the $N = 120$ data vector X_t into a block of slow-moving' variables that are largely predetermined, and another consisting of 'fast moving' variables that are sensitive to contemporaneous news. The idiosyncratic errors are assumed to be serially uncorrelated.

BBE Identification

1 Estimate F_t.

  i Let $\widehat{C}(F_t, R_t)$ be the K principal components of X_t.

  ii Let X_t^S be N_S 'slow' moving variables that do not respond immediately to a monetary policy shock. Let the K principal components of X_t^S be $C^\star(F_t)$.

  iii Define $\widehat{F}_t = \widehat{C}(F_t, R_t) - \widehat{b}_R R_t$ where $\widehat{b}_R$ is obtained by least squares estimation of the regression

$$\widehat{C}(F_t, R_t) = b_C C^\star(F_t) + b_R R_t + e_t.$$

2 Estimate the loadings by regressing X_t on $\widehat{F}_t$ and R_t: $\widehat{\Lambda^F}$ and $\widehat{\Lambda^R}$.

3 Estimate the FAVAR given by (23) and let $\widehat{\eta}_t$ be the residuals. From the triangular decomposition of the covariance of $\widehat{\eta}_t$, let A_0 be a lower triangular matrix with ones on the main diagonal Then $\widehat{\eta}_t = \widehat{A}_0 \widehat{\varepsilon}_t$ are the monetary policy shocks.

4 Obtain IRFs for $\widehat{F}_t$ and R_t by inverting (23) and using $\widehat{A}_0$

5 Multiplying the IRFs from the previous step by $\widehat{\Lambda^F}$ and $\widehat{\Lambda^R}$ to obtain the IRFs for X_t.

The novelty of the BBE analysis is that Step (1) accommodates the observed factor R_t when $\widehat{F}_t$ is being estimated. By construction, $\widehat{C}(F_t, R_t)$ spans the space spanned by F_t and R_t while $C^*(F_t)$ spans the space of common variations in variables that do not respond contemporaneously to monetary policy. Since R_t is observed, the regression then constructs the component of $\widehat{C}_t$ that is orthogonal to R_t. Once $\widehat{F}_t$ is available, Step (2) is straightforward. Under the BBE scheme, the common shocks are identified in Step (3) when a FAVAR in $(\widehat{F}_t, R_t)$ is estimated. Because $\widehat{F}_t$ may be correlated contemporaneously with R_t, the monetary policy shocks are identified by ordering R_t after $\widehat{F}_t$ in (23).[12]

The lower triangular of A_0 is not enough to identify the structural shocks as the response depends on the product $\begin{pmatrix}\Lambda^F & \Lambda^R\end{pmatrix} A_0$.[13] Thus, BBE impose additional restrictions. In particular, the K slow moving variables are ordered first in X_t. Furthermore, the $K \times K$ block of Λ^F is identity, and the first element in Λ^R is zero. As a result, the first $K+1 \times K+1$ part of the product $\begin{pmatrix}\Lambda^F & \Lambda^R\end{pmatrix} A_0$ is lower triangular. For $K = 2$,

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \lambda_{31} & \lambda_{32} & \lambda_{33} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ a_{21} & 1 & 0 \\ a_{31} & a_{32} & 1 \end{pmatrix}$$

The structural model is just-identified.

Stock and Watson (2005) The SW approach treats monetary policy as a dynamic factor. The identification assumptions are that (i) the monetary policy shock does not affect the slow-moving variables contemporaneously; and (ii) the slow-moving shock and monetary policy affects the Fed Funds rate contemporaneously. Thus, as in Bernanke, Boivin, and Eliasz (2005), the slow-moving variables are ordered first, followed by the Fed funds rate, and then the fast-moving variables. The point of departure is that $\varepsilon_{Xt} = \Lambda \varepsilon_{ft} + v_{Xt}$ is assumed to have a factor structure and $\varepsilon_{ft} = G\eta_t = GBv_{ft}$. Letting $C = GB$, the errors are related by

$$\varepsilon_{Xt} = \Lambda C v_{ft} + v_{Xt}$$

where v_{ft} is of dimension q. The steps are as follows:

1 Let $\widehat{\varepsilon}_{Xt}$ into $(\widehat{\varepsilon}^{slow}_{X,t}$, $\widehat{\varepsilon}^{fed}_{X,t}$ and $\widehat{\varepsilon}^{fast}_{X,t})$ corresponding to the three types of variables.

2 Let $\widehat{u}_{Ft}$, the residuals from a VAR in the static factors constructed from the full panel, X.

3 Let the factor component of $\varepsilon^{fed}_{X,t}$ be the fit from a reduced rank regression of $\widehat{\varepsilon}^{slow}_{X,t}$ and $\widehat{u}_{Ft}$.

4 Take the monetary shocks to be the residuals from a projection of of $\widehat{\varepsilon}^{fed}_{X,t}$ onto $\widehat{v}^{slow}_{X,t}$.

If there are q^{slow} and q^{fast} factors in $\widehat{\varepsilon}^{slow}_{Xt}$ and $\widehat{\varepsilon}^{fast}_{Xt}$ respectively, then $q = q^{slow} + q^{fast} + 1$. The identification scheme makes use of the fact that $v^{slow}_{X,t}$ spans the space of $\varepsilon^{slow}_{X,t}$ and can thus be

[12] Boivin, Giannoni, and Stevanović (2010) use an alternative way to estimate F_t that does not rely on organizing the variables into fast and slow.

1 Initialize $\widehat{F}_t$ to be the K first principal components of X_t.

2 (i) Regress X_t on $\widehat{F}_t$ and R_t, to obtain $\widehat{\Lambda}^{F,j}_t$ and $\widehat{\Lambda}^{R,j}_t$. (ii) Compute $\widetilde{X}^j_t = X_t - \widehat{\Lambda}^{R,0}_t R_t$ (iii) Update $\widehat{F}_t$ as the first K principal components of $\widetilde{X}_t$

By construction, $\widehat{F}_t$ is contemporaneously uncorrelated with R_t This is possible because the step that estimates the latent factors controlling for the presence of the observed factors is separated from identification of structural shock. In BBE, η_t depends on the choice of variables used in the first stage to estimate F_t.

[13] In BBE application, Step 2 estimates of the loadings of slow moving variables to R_t are close to zero.

identified from a projection of $\widehat{\varepsilon}^{slow}_{X,t}$ on $\widehat{u}_{Ft}$. An additional step is needed to estimate the common variations between $\widehat{u}_{Ft}$ and $\widehat{\varepsilon}^{slow}_{X,t}$. This procedure sequentially estimates the rotation matrix B[14]. Note that the identification restrictions are imposed directly on the impact coefficients matrix of the structural moving average representation of X_t, and the structural model is overidentified. The method is not easily generalizable to other models in which the shocks do not have a block recursive structure implicit in the model.

FGLR: Forni, Giannone, Lippi, and Reichlin (2009) provides a framework for structural FAVAR analysis. The method is applied to identify monetary policy shocks in Forni and Gambetti (2010).

1 let $\widehat{\Lambda}$ be a $N \times r$ matrix of estimated loadings and $\widehat{F}_t$ be the static principal components. Estimate a VAR in $\widehat{F}_t$ to get $\widehat{\Gamma}(L)$ and the residuals $\widehat{u}_{Ft}$.

2 Perform a spectral decomposition of the covariance matrix of $\widehat{u}_{Ft}$. Let M be a diagonal matrix consisting of the largest eigenvalue of $\widehat{u}_F \widehat{u}'_F$ and let $\mathcal{K}$ be the $r \times q$ matrix of eigenvectors.

3 Let $S = \mathcal{K}M$. The non-orthogonalized impulse responses are given by

$$\ddot{\Psi}^{\eta}(L) = \widehat{\Lambda}(I - \widehat{\Gamma}(L))^{-1}S.$$

Step (2) is a consequence of the fact that the VAR in $\widehat{F}_t$ is singular. Step (3) rotates $\ddot{\Psi}^{\eta}$ by a $q \times q$ matrix of restrictions. Unlike the partial identification analysis of Stock and Watson (2005), this method estimates the impulse responses for the system as a whole. Mis-specification in a sub-system can affect the entire analysis, but the estimates are more efficient if every aspect of the factor model is correctly specified.

[14] Boivin, Giannoni, and Stevanović (2013) find that the rotation of principal components by $\widehat{B}$ gives interpretable factors.

Table 1: Simulation Experiment I, DGP 1

	$corr(v_{ft}, \widehat{v}_{ft})$							
	a. Recursive Ordering				b. Block Ordering			
DGP 1a	$\begin{pmatrix} 0.9844 & 0.0308 \\ 0.0636 & 0.9790 \end{pmatrix}$				$\begin{pmatrix} 0.9825 & 0.0562 \\ 0.0953 & 0.9030 \end{pmatrix}$			
DGP 1b	$\begin{pmatrix} 0.9843 & 0.0313 \\ 0.0638 & 0.9777 \end{pmatrix}$				$\begin{pmatrix} 0.9805 & 0.0874 \\ 0.1188 & 0.8706 \end{pmatrix}$			
	RMSE of Impulse Responses: v_{ft} observed							
	$v_{f1} \to X_1$	$v_{f2} \to X_1$	$v_{f1} \to X_N$	$v_{f2} \to X_N$				
DGP 1a	0.0415	0.0412	0.0426	0.0414				
DGP 1b	0.0409	0.0394	0.0422	0.0425				
	RMSE of Impulse Responses Using $\widehat{v}_{ft}$							
	a. Recursive Ordering				b. Block Ordering			
	$v_{f1} \to X_1$	$v_{f2} \to X_1$	$v_{f1} \to X_N$	$v_{f2} \to X_N$	$v_{f1} \to X_1$	$v_{f2} \to X_1$	$v_{f1} \to X_N$	$v_{f2} \to X_N$
DGP 1a	0.2452	0.2228	0.2299	0.1963	0.2558	0.2642	0.2366	0.2032
DGP 1b	0.2994	0.2453	0.2702	0.1953	0.2954	0.3556	0.2556	0.2578

This Table presents a summary of results from the simulation experiment I, DGP 1. It shows the correlation between the true and the estimated common shocks, the Root Mean Squared Error (RMSE) of impulse responses from the infeasible FADL regression when v_{ft} is observed, and the RMSE from the feasible FADL regression when $\widehat{v}_{ft}$ is used instead. X_1 is the 'fast' moving series that responds on impact to both shocks, while X_N the 'slow' variable that does not respond on impact to v_{f2}.

Table 2: Simulation Experiment I, DGP 2

	$corr(v_{ft}^{mp}, \widehat{v}_{ft}^{mp})$			
	a. Recursive Ordering		b. Block Ordering	
DGP 2a	0.9747		0.9629	
DGP 2b	0.9774		0.9620	
	RMSE of Impulse Responses: v_f Observed			
	$v_{f2} \to X_1$	$v_{f2} \to X_N$		
DGP 2a	0.0406	0.0417		
DGP 2b	0.0399	0.0419		
	RMSE of Impulse Responses: v_f Estimated			
	a. Recursive Ordering		b. Block Ordering	
	$v_{f2} \to X_1$	$v_{f2} \to X_N$	$v_{f2} \to X_1$	$v_{f2} \to X_N$
DGP 2a	0.2743	0.2285	0.2760	0.1996
DGP 2b	0.3155	0.2781	0.3391	0.2466

This Table presents a summary of results from the simulation experiment I, DGP 2. It shows the results only with respect to the shock of interest: v_{ft}^{mp}.

Table 3: Simulation Experiment II, Aggregate series

	M1	M2	M3	M4	M1-ss	M2-ss	M3-ss	M4-ss
RMSE: MEAN								
consumption	1.578	1.478	1.844	1.583	1.203	**1.113**	1.432	1.265
inflation	0.613	**0.603**	0.620	0.653	0.753	0.747	0.739	0.752
output	1.798	1.637	1.967	1.766	1.339	**1.214**	1.473	1.371
hours	2.179	2.049	2.486	1.874	1.508	1.411	1.930	**1.389**
RMSE: MEDIAN	M1	M2	M3	M4	M1-ss	M2-ss	M3-ss	M4-ss
consumption	0.733	0.658	0.727	0.725	0.556	0.544	0.571	**0.524**
inflation	0.500	**0.485**	0.501	0.526	0.581	0.575	0.618	0.617
output	1.057	0.916	0.982	1.004	0.740	**0.696**	0.752	0.706
hours	1.320	1.213	1.373	1.159	0.837	**0.773**	1.056	0.779
RMAE: MEAN	M1	M2	M3	M4	M1-ss	M2-ss	M3-ss	M4-ss
consumption	1.142	1.095	1.174	1.130	0.945	**0.899**	1.032	0.982
inflation	0.643	**0.612**	0.646	0.717	0.775	0.744	0.745	0.803
output	1.295	1.222	1.299	1.262	1.050	**0.990**	1.105	1.071
hours	1.495	1.437	1.547	**1.380**	1.198	1.148	1.351	1.149
RMAE: MEDIAN	M1	M2	M3	M4	M1-ss	M2-ss	M3-ss	M4-ss
consumption	0.894	0.839	0.858	0.867	0.707	**0.692**	0.766	0.720
inflation	0.604	**0.582**	0.604	0.680	0.723	0.698	0.696	0.755
output	1.085	1.033	1.075	1.064	0.847	**0.807**	0.883	0.865
hours	1.269	1.210	1.279	1.189	0.986	0.946	1.139	**0.945**

This table presents a summary of results from the simulation experiment II. Numbers are relative to FAVAR: if < 1, FADL produces smaller error, in terms of the mean (median) of RMSE and RMAE, than FAVAR model. Bold characters correspond to the smallest error. The models M1 to M4-ss are summarized in Table 4.

Table 4: List of FADL models in simulation experiment II

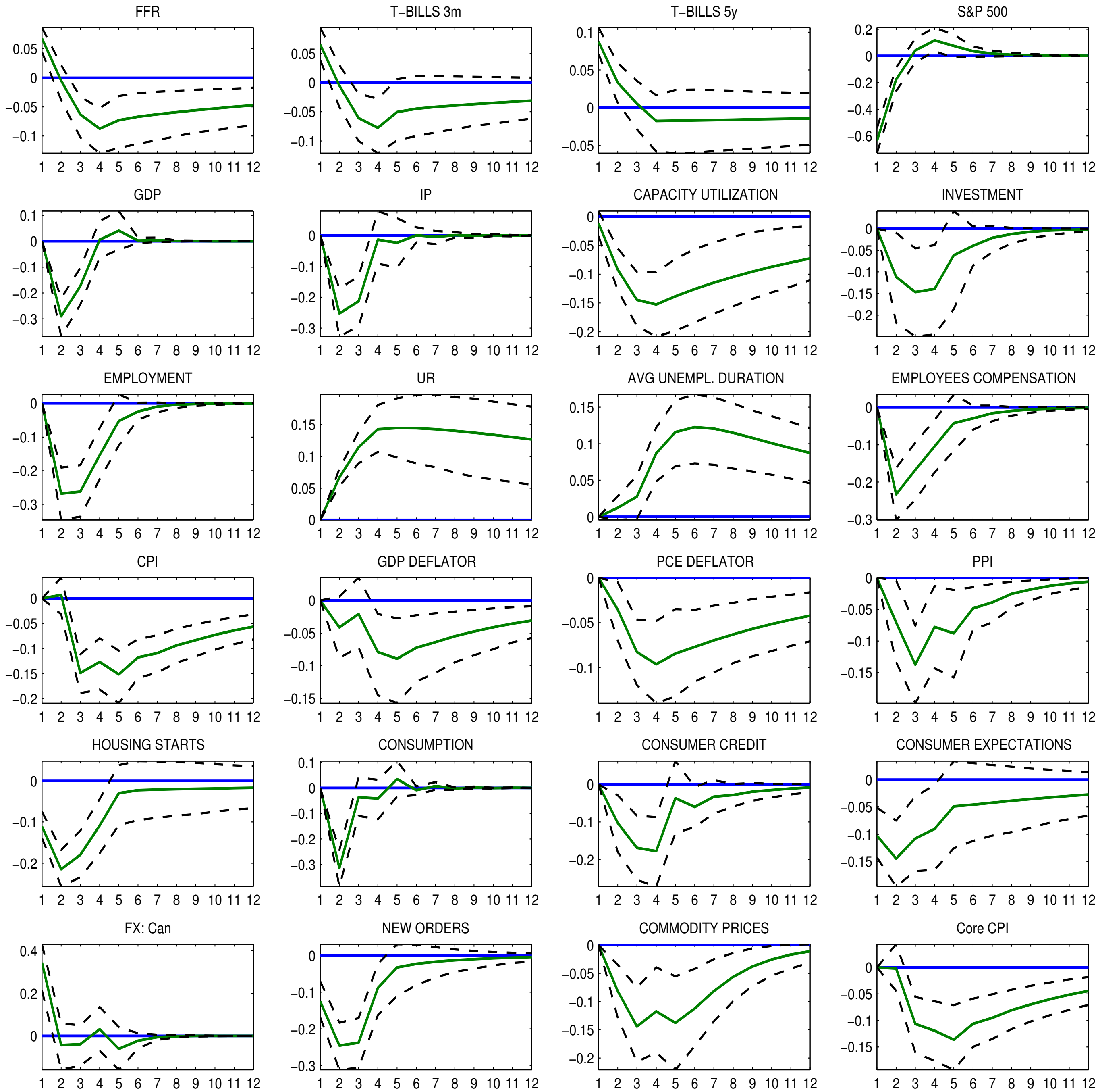
M1:	unrestricted FADL model (estimated number of common shocks *and* estimated lag structure)
M2:	fixed-lag FADL model (estimated q but fixed number of lags: 5 AR lags, 5 lags for each shock)
M3:	$q = 1$ FADL model (one latent common shock plus the money growth *and* estimated lag structure)
M4:	$q = 2$ FADL model (two latent common shock plus the money growth *and* estimated lag structure)
M1-ss:	same as M1 but common shocks estimated using a subset of series (118 instead of 156)
M2-ss:	same as M2 but common shocks estimated using a subset of series (118 instead of 156)
M3-ss:	same as M3 but common shocks estimated using a subset of series (118 instead of 156)
M4-ss:	same as M4 but common shocks estimated using a subset of series (118 instead of 156)

Table 5: News Shock, p-values from Wald test for $H_0 : \alpha_{yf}(L) = 0$

Variables in (X_t^{TFP}, X_t^{SP})	RO	BO	BO2	BO3
TFP	0	0	0	0
TFP-util	0	0	0	0
TFP-I	0	0	0	0
TFP-C	0	0	0	0
TFP-I-util	0	0	0	0
TFP-C-util	0	0	0	0
S&P: composite	0	0	0	0
S&P: industrial	0	0	0	0
S&P: dividend	0	0	0	0
S&P: price/earning	0	0	0	0
DJ: industrial	0	0	0	0
DJ: composite	0	0	0	0
DJ: transportation	0	0	0	0
DJ: utilities	0	0	0	0
Variables in X_t^{OTH}				
GDP	0	0	0	0
IP	0	0	0	0
CONSUMPTION	0	0	0	0
INVESTMENT	0	0	0	0
HOURS	0	0	0	0
HOURS: Overtime	0	0	0	0
EMPLOYEES: NONFARM	0,0262	0,0399	0	0
CLF: EMPLOYED	0	0	0	0
HELP-WANTED ADV	0	0	0	0
AVG HOURLY EARNINGS	0	0	0	0
CAPACITY UTILIZATION	0	0	0	0
CONSUMER EXPECTATIONS	0	0	0	0
UR	0	0	0	0
EMPLOYEES ROMPENSATION	0	0	0	0
CONSUMPTION: NONDUR	0	0	0	0
CONSUMPTION: DURAB	0	0	0	0
CONSUMER CREDIT	0,2422	0,2772	0	0
HOUSING STARTS	0,0062	0,0027	0	0
NEW ORDERS	0	0	0	0
INVENTORIES	0,0718	0,1742	0	0
FFR	0,3513	0,2612	0	0
COMMODITY PRICES	0,0299	0,1384	0	0
CPI	0,3527	0,5556	0	0
GDP DEFLATOR	0,0438	0,0729	0	0

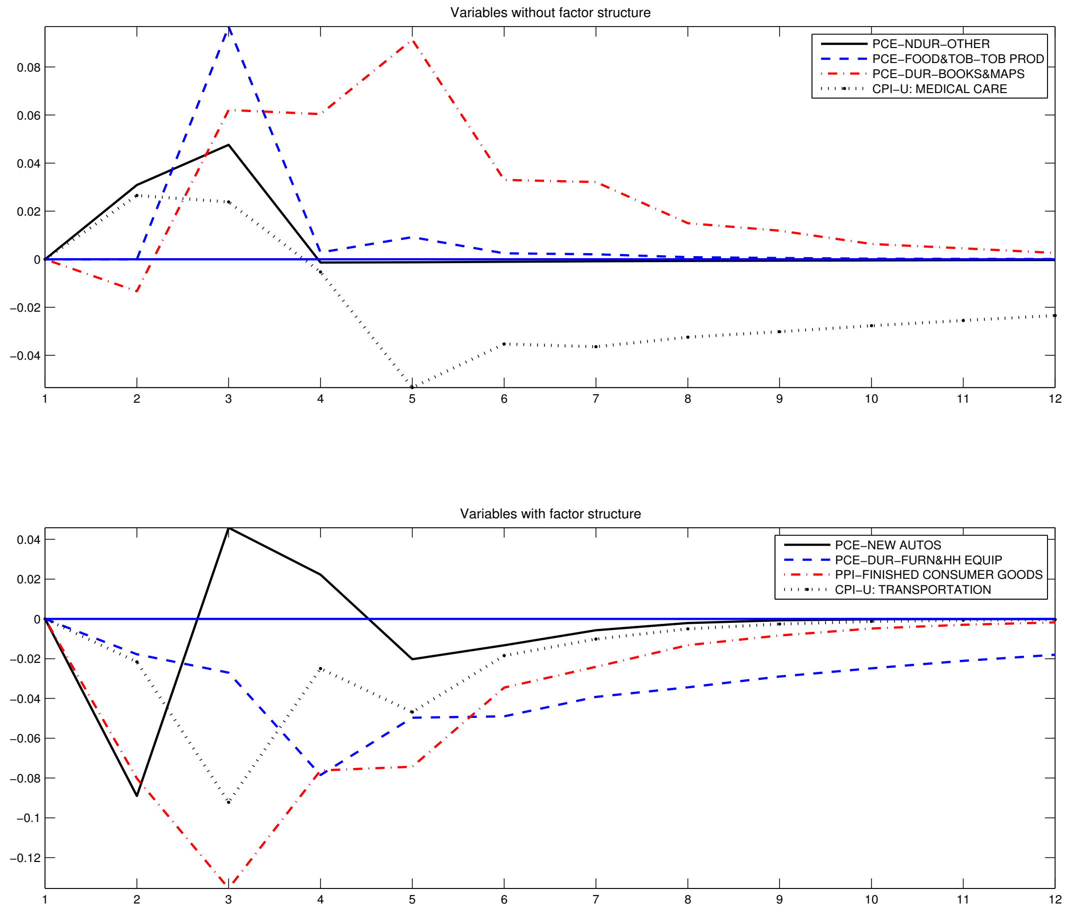
RO and BO refer to identification by causal and block ordering, respectively. The two blocks of data are X_t^{TFP} and X_t^{SP}. Data from the macro block X_t^M are also used in BO2 and BO3. BO2 uses the ordering TFP, SP, Macro. BO3 orders the macro variables before the stock prices.

Figure 1: Dynamic responses to a contractionary monetary policy shock



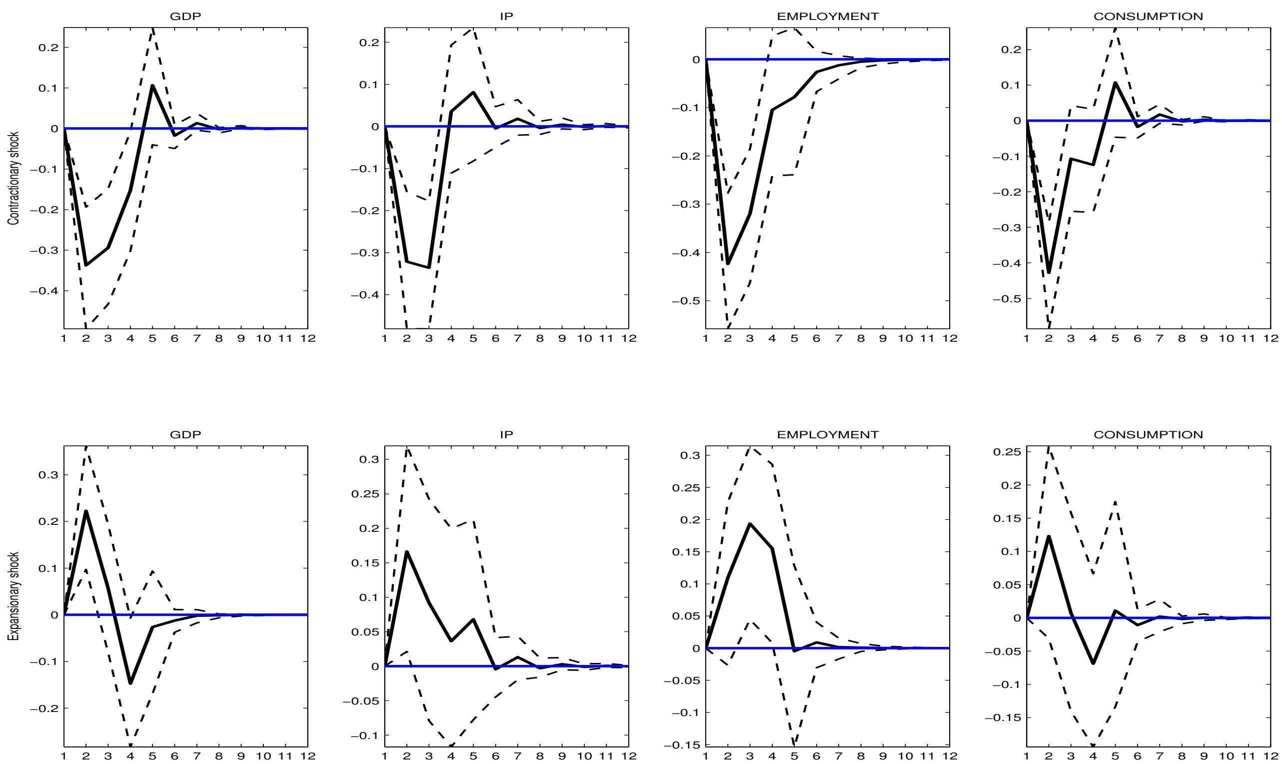
This Figure presents the FADL estimated impulse responses, together with 90% bootstrap confidence bands, of selected variables from X_t to a contractionary monetary policy shock as identified by recursive ordering assumption (RO) in Example 1.

Figure 2: Dynamic responses to a monetary policy shock of series in X_t^{OTH}



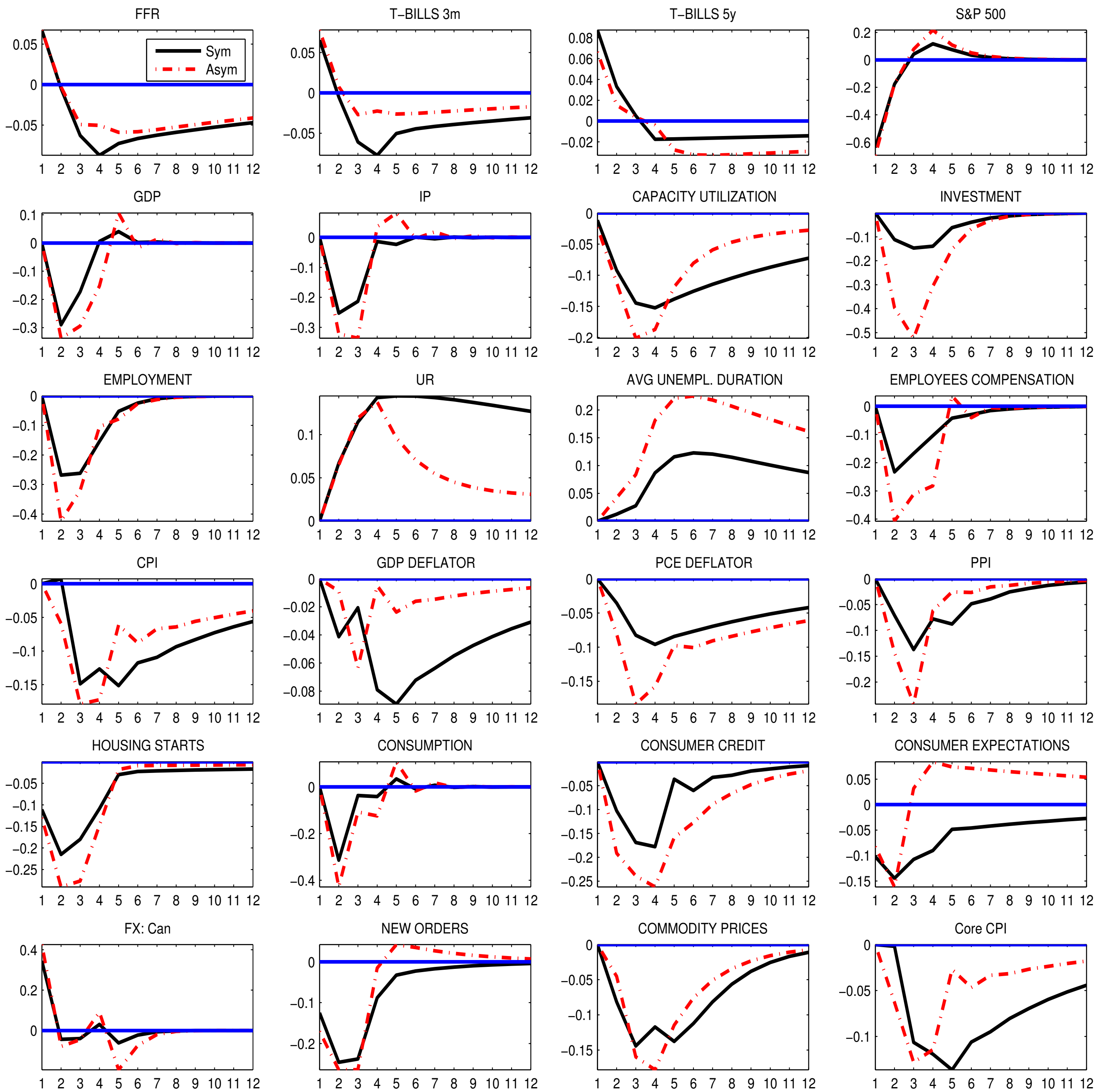
This Figure presents the FADL estimated impulse responses of selected disaggregated price indices from X_t^{OTH} *to a contractionary monetary policy shock as identified by recursive ordering assumption (RO) in Example 1. The top panel contains responses of series that do not load significantly on* $\widehat{v}_{ft}$*, while variables in the bottom panel do have significant distributed lag coefficients.*

Figure 3: Asymmetric IRFs to contractionary and expansionary monetary policy shocks



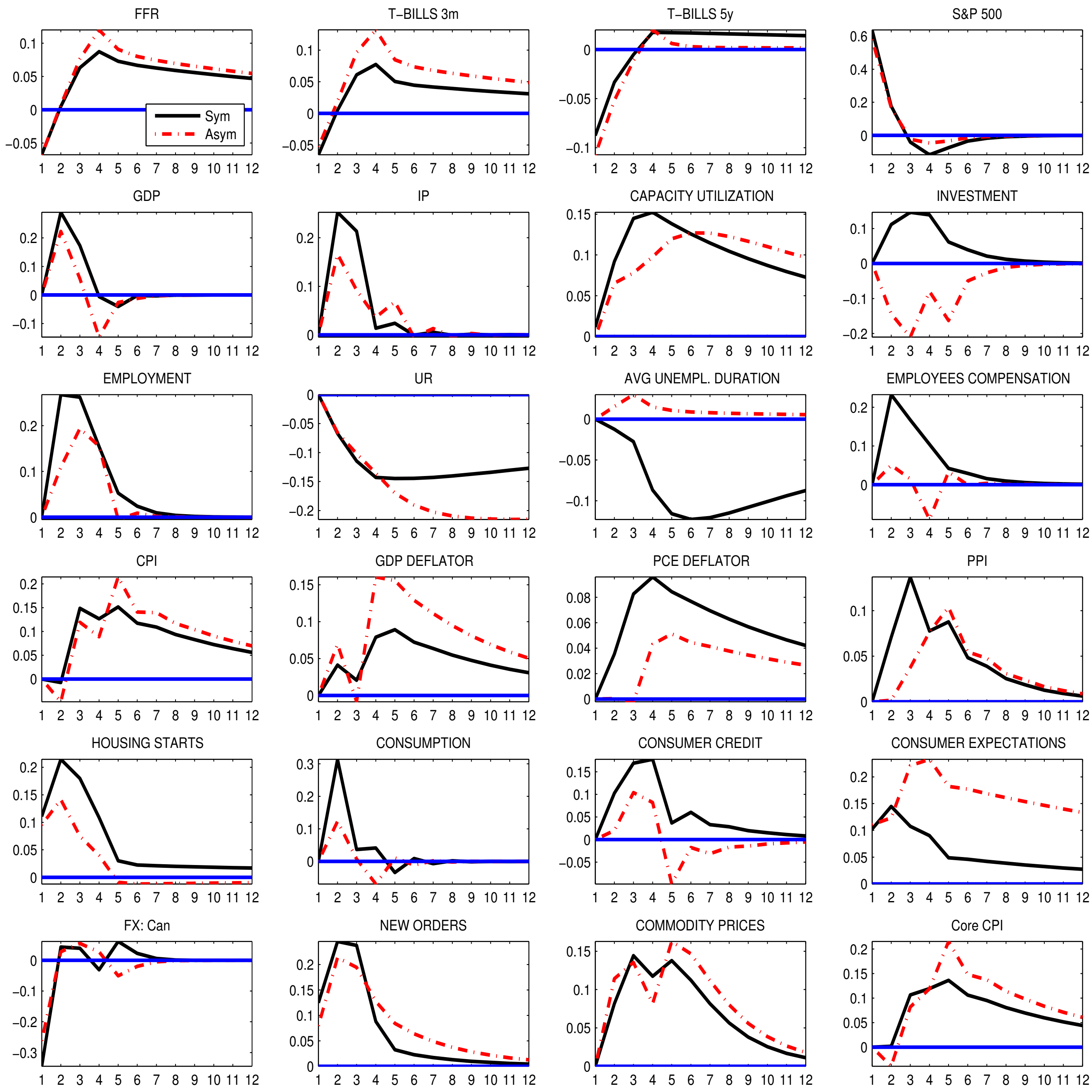
This Figure presents the FADL estimated asymmetric impulse responses, together with 90% *bootstrap confidence bands, of selected variables from* X_t *to* contractionary and expansionary *monetary policy shocks as identified by recursive ordering assumption (RO) in Example 1. The top 4 graphs show IRFs to a contractionary shock while the bottom graphs present IRFs to an expansionary monetary policy shock.*

Figure 4: Comparison of symmetric and asymmetric IRFs to a contractionary monetary policy shock



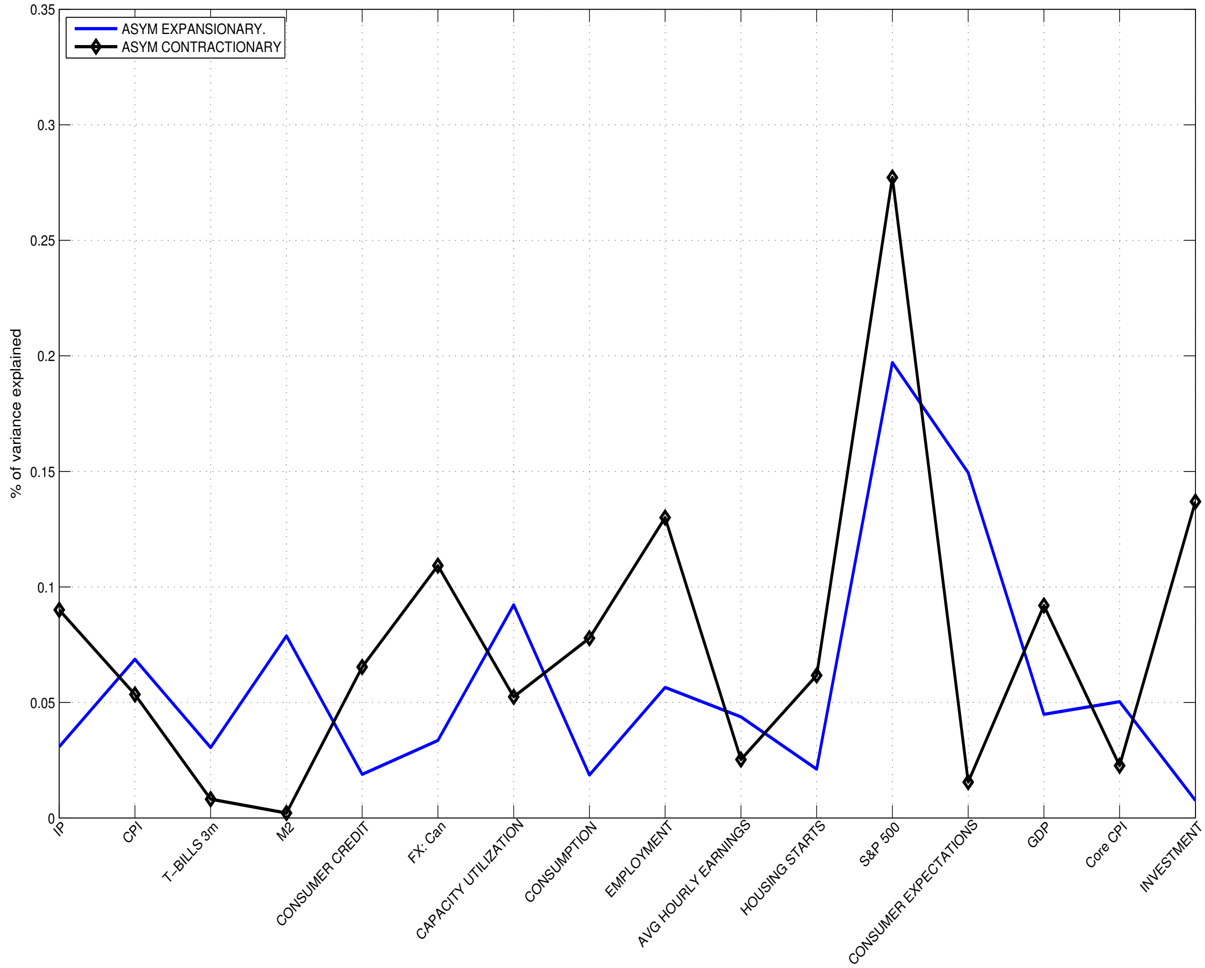
This Figure compares the FADL estimated symmetric *(full line) and* asymmetric *(dotted line) impulse responses of selected variables from X_t to a contractionary monetary policy shock as identified by recursive ordering assumption (RO) in Example 1.*

Figure 5: Comparison of symmetric and asymmetric IRFs to an expansionary monetary policy shock



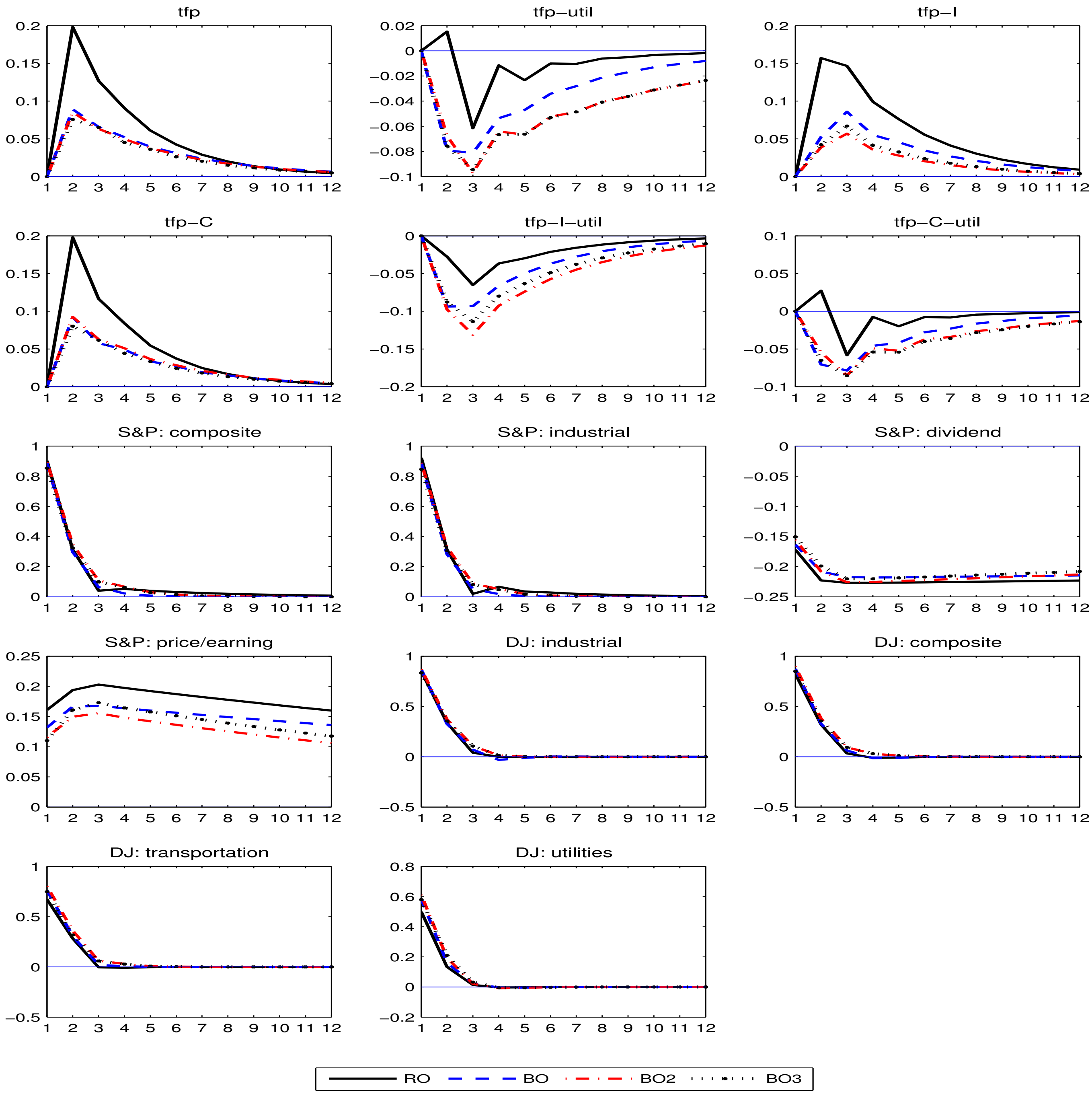
This Figure compares the FADL estimated symmetric *(full line) and* asymmetric *(dotted line) impulse responses of selected variables from* X_t *to an expansionary monetary policy shock as identified by recursive ordering assumption (RO) in Example 1.*

Figure 6: Variance decomposition after asymmetric expansionary and contractionary monetary policy shocks



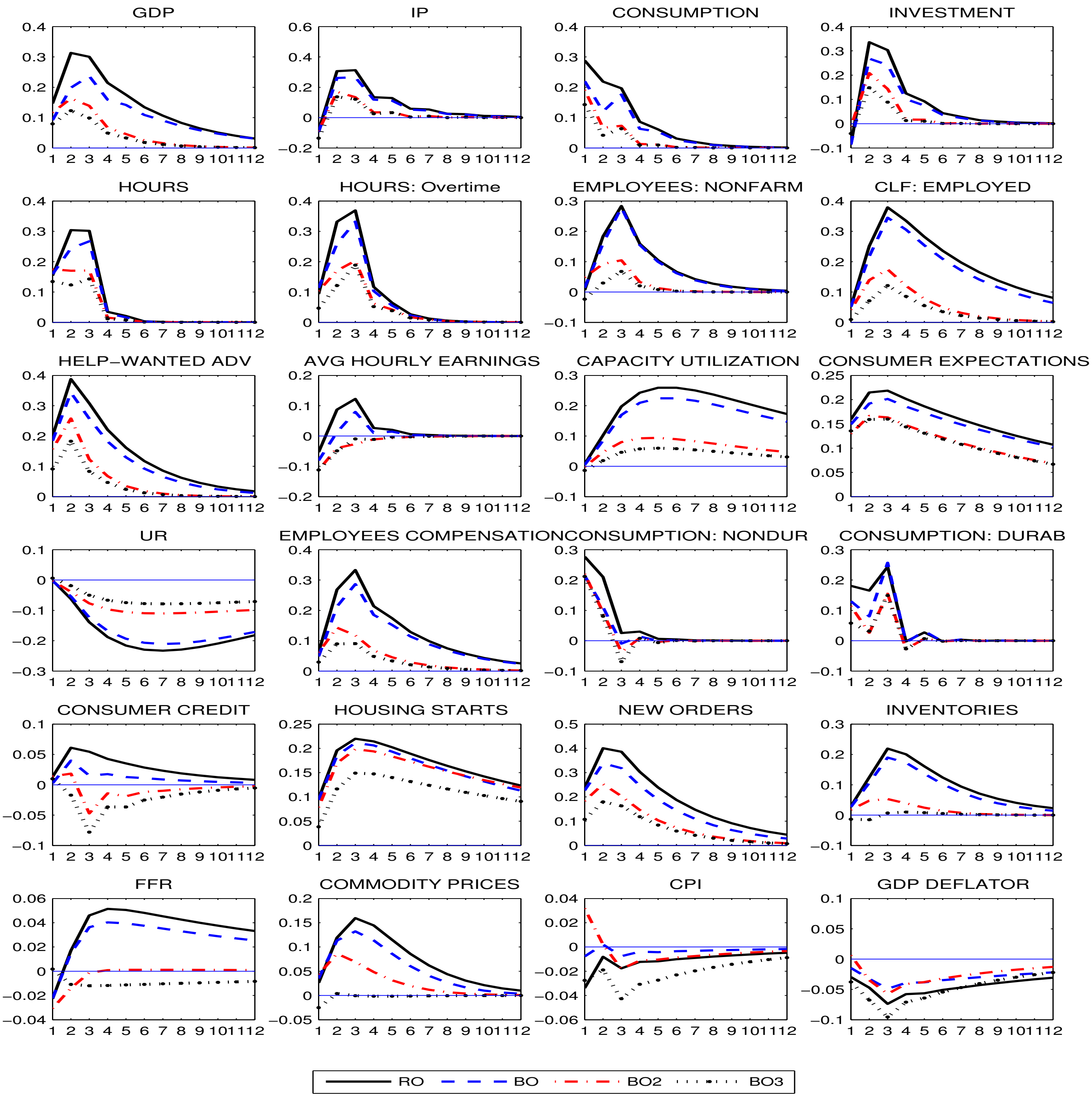
This Figure plots the FADL estimate of the variance decomposition of selected variables from X_t after expansionary *(blue line) and* contractionary *(black line) asymmetric monetary policy shocks as identified by recursive ordering assumption (RO) in Example 1.*

Figure 7: Dynamic responses of X_t^{TFP} and X_t^{SP} to a positive *news* shock



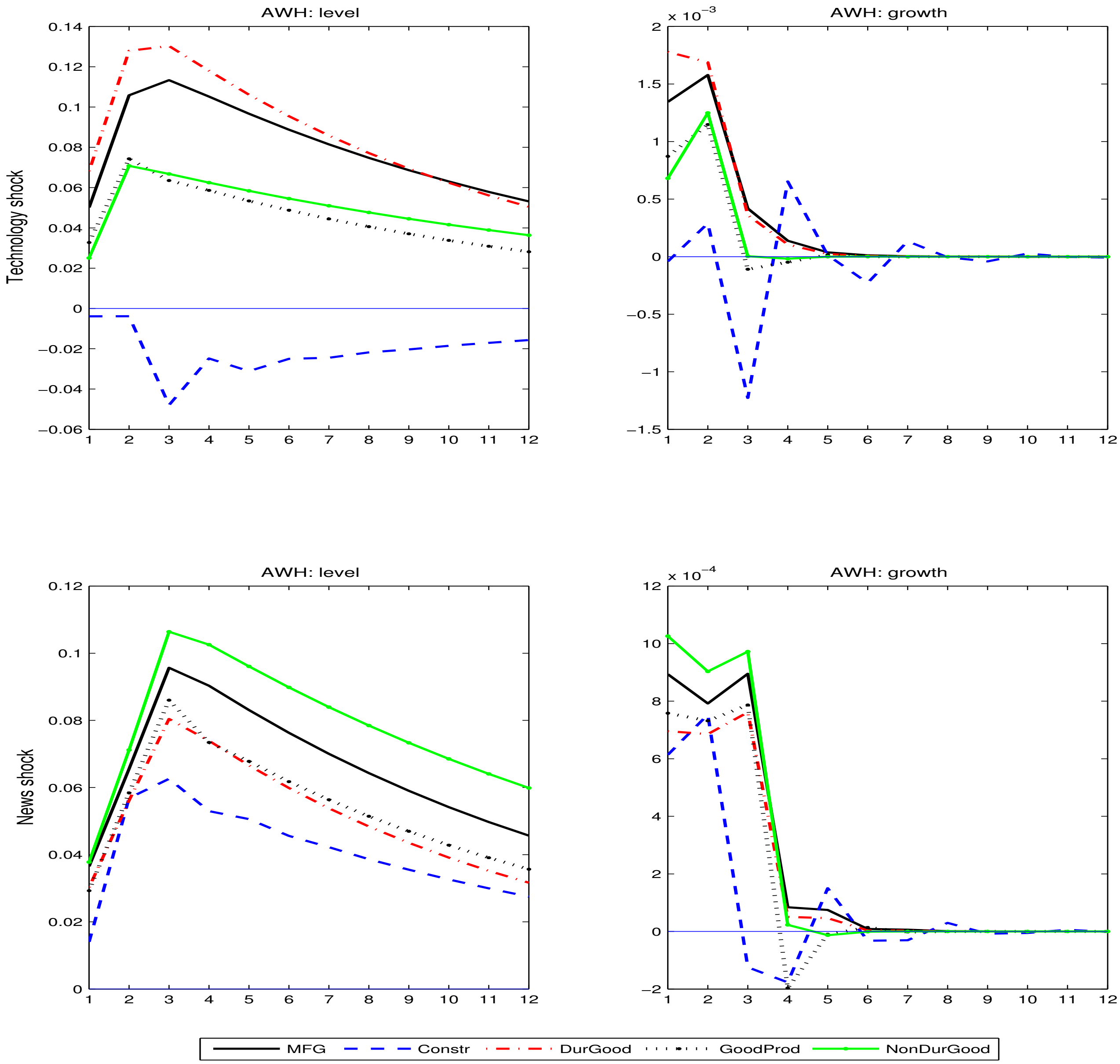
This Figure presents the FADL estimated impulse responses of all variables in X_t^{TFP} and X_t^{SP} to a positive news *shock as identified by recursive ordering assumption (RO) and block ordering assumptions (BO, BO2, BO3) in Example 2.*

Figure 8: Dynamic responses of selected series in X_t^{OTH} to a positive *news* shock



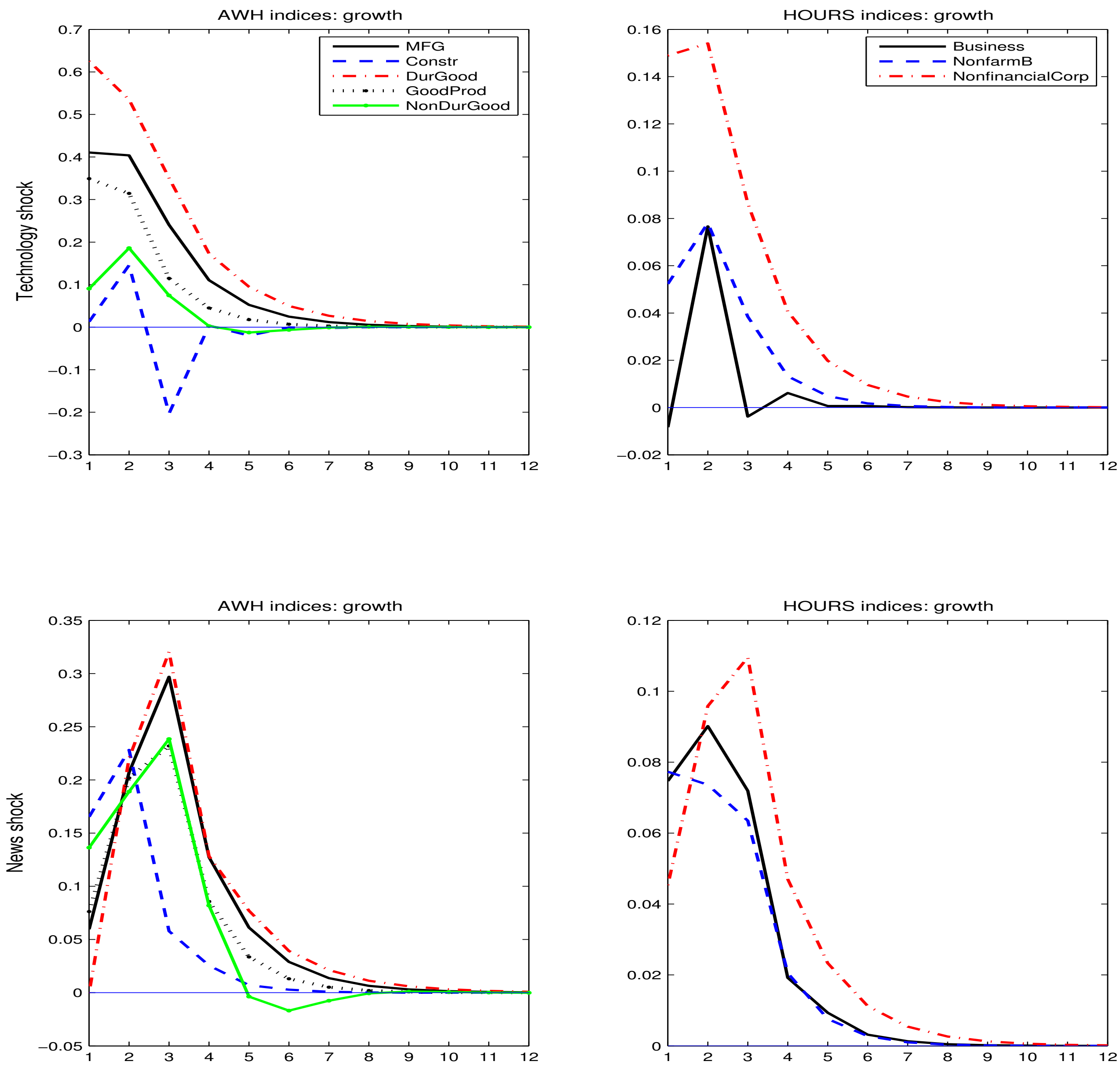
This Figure presents the FADL estimated impulse responses of selected variables in X_t^{OTH} to a positive news *shock as identified by recursive ordering assumption (RO) and block ordering assumptions (BO, BO2, BO3) in Example 2.*

Figure 9: Dynamic responses of average weekly hours measures to a positive *news* shock



This Figure presents the FADL estimated impulse responses of average weekly hours (AWH), in level and in growth, to a positive news *shock as identified by block ordering assumptions (BO3) in Example 2.*

Figure 10: Dynamic responses of hours, measured as indices, to a positive *news* shock



This Figure presents the FADL estimated impulse responses of selected sectoral average weekly hours indices (AWH indices) and total hours indices (HOURS indices), in growth, to a positive news *shock as identified by block ordering assumptions (BO3) in Example 2.*

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